STUDY OF MODEL MATCHING TECHNIQUES FOR THE DETERMINATION OF PARAMETERS IN HUMAN PILOT MODELS

REPORT ON TASK 1

LINEAR, TIME-INVARIANT MODELS

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ABSTRACT

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This report describes the results of an investigation of automatic computer techniques for determining the parameters of human pilot models in tasks where the pilot's performance can be described by a linear time-invariant model. This work constitutes Task 1 of a study of model matching techniques being conducted under NASA Contract NAS1-2582.

Several model matching strategies including continuous and discontinuous, iterative schemes have been developed, analyzed, investigated experimentally, and their performance and computer implementation have been evaluated. Relative merits of these different methods have been explored, and guide lines for future study have been established to determine preferred model matching approaches.

The methods reported here have been applied successfully to the determination of human pilot parameters in a single axis compensatory tracking task. Results are compared with those previously reported in the literature. Satisfactory agreement of data obtained from the different model matching techniques has been demonstrated. Author

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1. INTRODUCTION

This report presents the results of the first phase of a research program concerned with the development of automatic computer techniques for the determination of parameters in mathematical models of human pilots. The major objective of the program is the development and evaluation of alternate techniques for optimization of human pilot models in a variety of situations, including those in which the pilot's performance may be characterized by time variation or nonlinearities. The overall philosophy of the research program is based on the utilization of so-called model matching techniques, i.e., techniques in which the performance of the model is compared with the performance of the human pilot and differences in performance are used to adjust model parameters in such a way that this difference is minimized. The first phase of the program has concentrated on the investigation of model matching techniques for the determination of human pilot models in tasks where the pilot's performance can be described by a linear invariant model. This report describes in detail the three computational strategies used for optimization of the models, describes the experimental situation, presents the results obtained from the application of both iterative and continuous model matching techniques to human tracking data, and compares these results with those obtained in the literature. Problems of computer implementation are discussed in the Appendix.

The task selected for phase 1 of this study consisted of compensatory tracking in one axis using a fingertip controller. Several simple controlled element dynamics were used. The forcing function consisted of low frequency random noise filtered by a third order filter with a break frequency at 1.0 radians/sec. An extensive literature on this type of tracking situation exists (References 2 and 3). The results of spectral analysis studies have shown that the performance of the human pilot in this type of task can be approximated closely by means of a linear differential equation with constant coefficients, usually called a quasi-linear describing function. Consequently, there is good justification for choosing a linear time invariant model for the representation of human pilot tracking performance in this task. The model-matching technique consists in comparing the outputs of the assumed linear model with the output of a human pilot, when both pilot and model are subjected to the same input. Several methods of determining the parameter adjustments based on the differences between model and pilot performance have been investigated.

In order to obtain meaningful comparisons with previous work, the mathematical model chosen to represent the pilot was similar to that used by Adams (Reference 4) and contained a second order denominator. The experimental design and apparatus employed in the study are described in Section 2. The particular model-matching techniques of parameter adjustment utilized in the study are described in Section 3. The results are presented in Section 4.

2. DESCRIPTION OF EXPERIMENTAL PROCEDURE

2.1 General Approach

The general philosophy of the model-matching approach is illustrated in Figure 1. It can be seen that the same input signal is applied both to the human operator and to a mathematical model. The outputs of model and operator are compared and an appropriate performance function is used as an input to a parameter adjustment strategy program, which in turn determines the parameter values. In order to make it possible to apply identical experimental data to several model-matching strategies it was necessary to record input signals and human operator responses in particular experimental situations, and then use recorded information as inputs to the computer studies. The experimental situation can be visualized conveniently with reference to Figure 2. The task chosen is a compensatory tracking task in one axis. In order to demonstrate the applicability of the methods two subjects were used, each performing three runs with each of three different controlled element configurations. While nine runs for each of two trained operators are not sufficient to establish the statistical characteristics of the experiment, it is an adequate sample for evaluating the feasibility of the parameter optimization methods.

2.2 Display and Control Configuration

The display utilized with this study was a 5-inch oscilloscope which displayed the tracking error as a vertical displacement of the spot from the center of the oscilloscope screen. The hand controller utilized for this study was a three-degree-of-freedom spring-centered controller, with a lever arm of approximately 4-1/2 inches. Since the tracking task was restricted to a single-axis, only one potentiometer on the hand controller was energized. The controller exhibits negligible inertia and damping

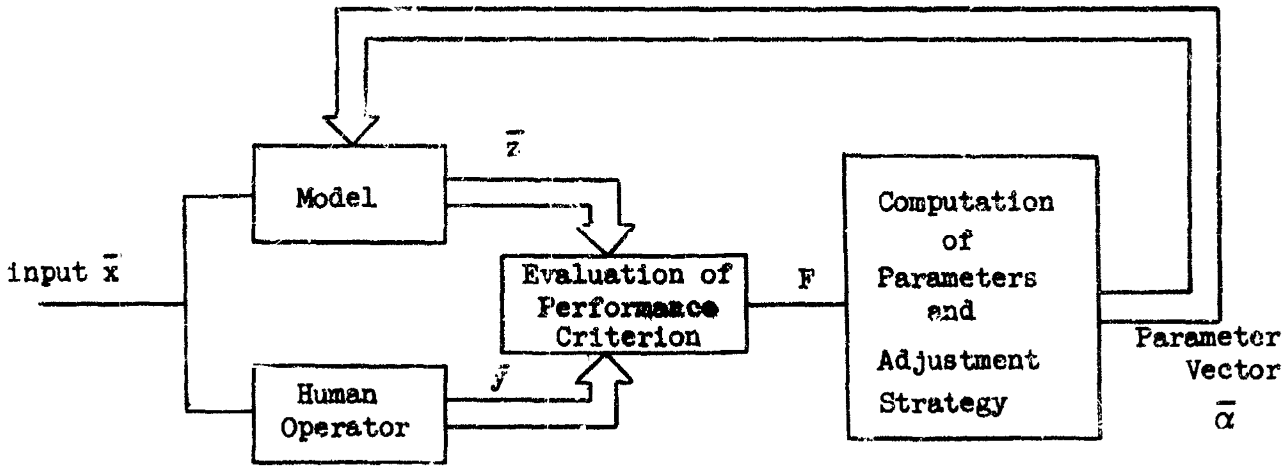


Figure 1.

Model Matching Control System

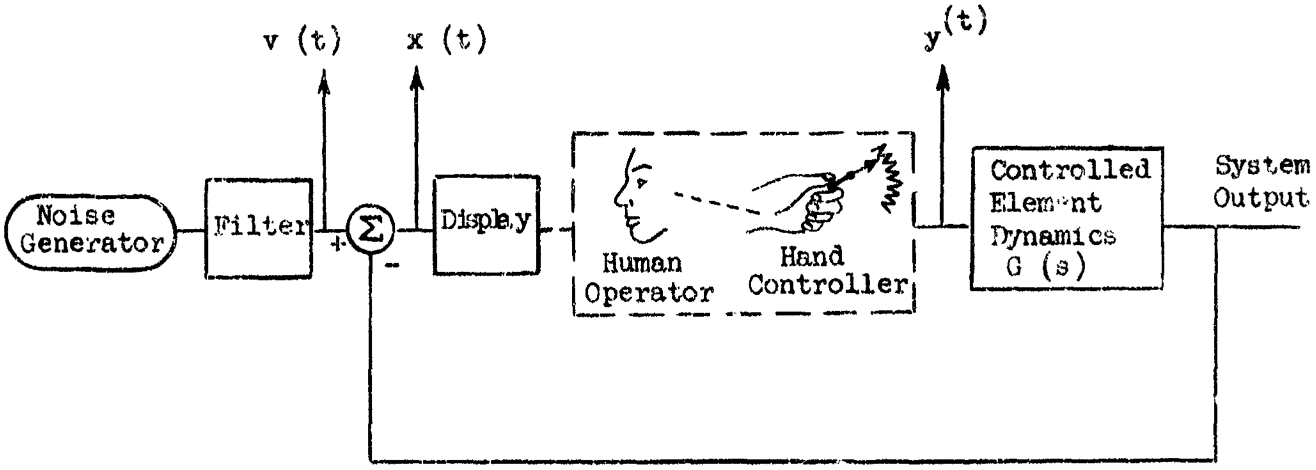


Figure 2

Tracking Task of the Human Operator

and can be considered as a position controller. During the experiment the pilot was seated in a simple fixed base cockpit simulator in order to be as free as possible from external disturbances. Figure 3 illustrates the cockpit used in the experiment.

2.3 Controlled Element Dynamics

Three simple controlled element configurations were simulated on an analog computer and utilized in the experiment. The specific configurations were

$$G_1(s) = \frac{K_1}{s(s+1)} \qquad (1)$$

$$G_2(s) = \frac{K_2}{s^2}$$

$$G_3(s) = \frac{K_3}{(s+1)}$$

2.4 Input Signal

Filtered low-frequency Gaussian noise was used as the input or disturbance signal v(t). The spectral density of the noise source can be considered flat for the frequency range of interest. The filter utilized is described by the transfer function

$$G_4(s) = \frac{K\,s}{(10s+1)(s+1)^3} \qquad (2)$$

Some of the parameter adjustment techniques utilized require the availability of both the input signal $x(t)$ and its first derivative $\dot{x}(t)$. An analog differentiator was employed to obtain the first derivative of the input signal.

2.5 Tape Recording

As mentioned above it was desired to use the same experimental data with different computational strategies, in order to avoid ambiguities which may be introduced by the variability of human pilot performance. Consequently, the pilot's input and output signals were recorded on a Precision Instrument Co. FM magnetic tape recorder, appropriately coded for each experiment. In addition, voltage pulses which could be used for control of the analog computer were recorded simultaneously on an adjacent track. The signals recorded on tape are detailed below in Table 1.

Table 1

Tape Recorded Information

Channel	
1	Display signal $x(t)$
2	Display rate $\dot{x}(t)$
3	Operator output $y(t)$
4	Operator output rate $\dot{y}(t)$
5	Input disturbance $v(t)$
6	Control signal
7	Identification marks

Each individual tracking run lasted five minutes. From the five-minute runs 30 second samples were selected and re-recorded on tape loops which could be used for iterative parameter adjustment, as described in the following section.

2.6 Computer Equipment

An analog computer was utilized for generating the input signals, simulating the controlled element, and driving the display during the experiment. In addition, analog computers were utilized for the parameter optimization process. The iterative computer optimization technique was instrumented on an iterative analog computer which has provisions for independent control of the operating modes on each integrator, as well as provision for the use of any integrator as a track-and-hold channel. The details of this computer implementation are discussed in the Appendix of this report.

3. MODEL-MATCHING TECHNIQUES OF PARAMETER IDENTIFICATION

3.1 Statement of the Problem

The methods of parameter optimization considered in this research are based on the comparison of performance of an assumed mathematical model with that of the human pilot, as indicated in Figure 1. An automatic parameter adjusting strategy must be selected which determines the optimum values of the model parameters, in the sense that the model performance approximates as closely as possible the human pilot performance. It can be seen that the mechanization of techniques of this type requires three primary considerations:

(a) A mathematical model must be selected and the adjustable parameters fully identified. In the present case it has been indicated above that there is good evidence for the selection of a second order differential equation as a model for the operator's performance.

(b) A performance function must be constructed, such that this performance function is an index of the validity of the mathematical model. Minimization of this error function or performance function by adjustment of model parameters results in the closest possible agreement between pilot performance and model performance.

(c) An automatic technique for performing the required parameter adjustments must be selected.

The purpose of this section is to present a brief discussion of the rationale which underlies the selection of the type of criterion function or performance function used in the experiments and to present three different adjustment strategies which were employed. The results obtained with these strategies will be given in the following section.

3.2 On the Choice of Performance Criterion

The criterion which compares the performance of system and model must be selected with considerable caution. Consider first the mathematical description of the model and pilot behavior. Dynamical systems are described by means of differential equations. Thus a system of order n will be described either by a single nth order differential equation or by n first order equations. Thus a model order n can be described by the set of equations

$$(3) \quad \dot{z}_i = f_i\,(z_1, z_2 \ldots\ldots z_u;\ t;\ \alpha_1, \alpha_2, \ldots\ldots \alpha_m)$$

$$i = 1, 2, \ldots n$$

where the z_i can be considered to be the corresponding time derivatives of the model output, that is

(4) $$z_i = \frac{d^i z}{dt^i}$$

and the α_j represent the adjustable parameters in the model. Time (t) appears explicitly in Equations 3 since the model output also depends on the time dependent input x. To completely characterize any dynamic system a set of initial condition is required in addition to the system equations and these are given by

(5) $$z_i(0) = c_i \qquad i = 1, 2, \ldots. n$$

where c_i represents the value of the ith derivative at the initial time. Since the set of variables z_i constitute a complete description of model behavior at any particular time, they are commonly considered as describing the state of the system at that time. In more concise mathematical terminology, the variables z_i are considered as components of a state vector $\bar{z}$ and are usually termed the state variables. Similarly the m adjustable parameters α_1 , $\alpha_2, \ldots \alpha_m$ can be considered the components of a parameter vector $\bar{\alpha}$. The set of first-order differential equations described by Equation 3 can then be stated as a single vector differential equation

(6) $$\dot{\bar{z}} = \bar{f}\,(\bar{z},\ t,\ \bar{\alpha}) \quad ; \quad \bar{z}(0) = \bar{c}$$

where $\bar{c}$ represents the initial state of the system. The model-matching problem consists in selecting a particular parameter vector $\bar{\alpha}$ such that the model behavior approximates as closely as possible the human pilot's behavior.

The state of the system to be identified, in this case the human pilot, can be denoted by means of the vector $\bar{y}$. However, whereas the order of the system represented by $\bar{z}$ is known since this is a model of assumed form, the last statement cannot be made about the system to be modeled, represented by the vector $\bar{y}$. In general, the system is of unknown order and may in fact be only partly deterministic. The problem of formulating a performance function then, is one of determining a distance between the vectors $\bar{z}$ and $\bar{y}$. In order to qualify as a distance function or metric in a function space, the criterion function must satisfy certain properties. A typical criterion function may be formulated as

$$(7) \quad F = \int_0^T \left[y_1(t) - z_1 (\bar{\alpha}, t) \right]^2 dt$$

where z_1 and y_1 represent the output positions of the model and pilot respectively. It is important to note that the criterion function F defined in Equation 7 is an ordinary function of the parameter vector $\bar{\alpha}$. That is, a selection of particular values for the parameters will result in a given number for the criterion function F upon evaluation of the definite integral in this equation. This formulation of the parameter optimization problem makes it possible to proceed on the basis of ordinary calculus which is concerned with the maximization or minimization of functions. On the other hand the criterion defined by

$$(8) \quad f = \left[y_1(t) - z_1 (\bar{\alpha}, t) \right]^2$$

depends not only on the parameter values but on the entire time history of the model output and consequently represents a functional whose maximization or minimization is the concern of the calculus of variations.

Consequently, it will be important to choose a performance function such as Equation 7 in order to facilitate the mathematical aspects of the problem. A more general form of Equation 7 is given by

$$(9) \quad F = \int_0^T \left\| \bar{y}(t) - \bar{z}(\bar{\alpha}, t) \right\|^2 dt$$

where the integrand represents the square of Euclidean norm and is defined by

$$(10) \quad \left\| \bar{x} \right\|^2 \triangleq x_1^2 + x_2^2 + x_3^2 + \ldots . \, x_n^2$$

Since F is an ordinary function of the parameters α_1, α_2, etc., it will be maximized or minimized when the gradient of F is equal to zero. Various methods of adjusting the parameters will then be based on the determination of a strategy which attempts to drive the performance criterion F toward its minimum value.

3.3 Possible Adjustment Strategies

The three parameter adjustment strategies employed during this study can be visualized conveniently with reference to the diagram of Figure 4 which illustrates contours of constant criterion function F in a parameter plane determined by the two adjustable parameters α_1 and α_2. The parameter optimization problem consists of beginning with an arbitrary initial set of values denoted by $\bar{\alpha}^{(0)}$, as indicated in the Figure, and proceeding automatically to the particular value of the parameter vector which results in the minimum possible value of the criterion function. Three possible

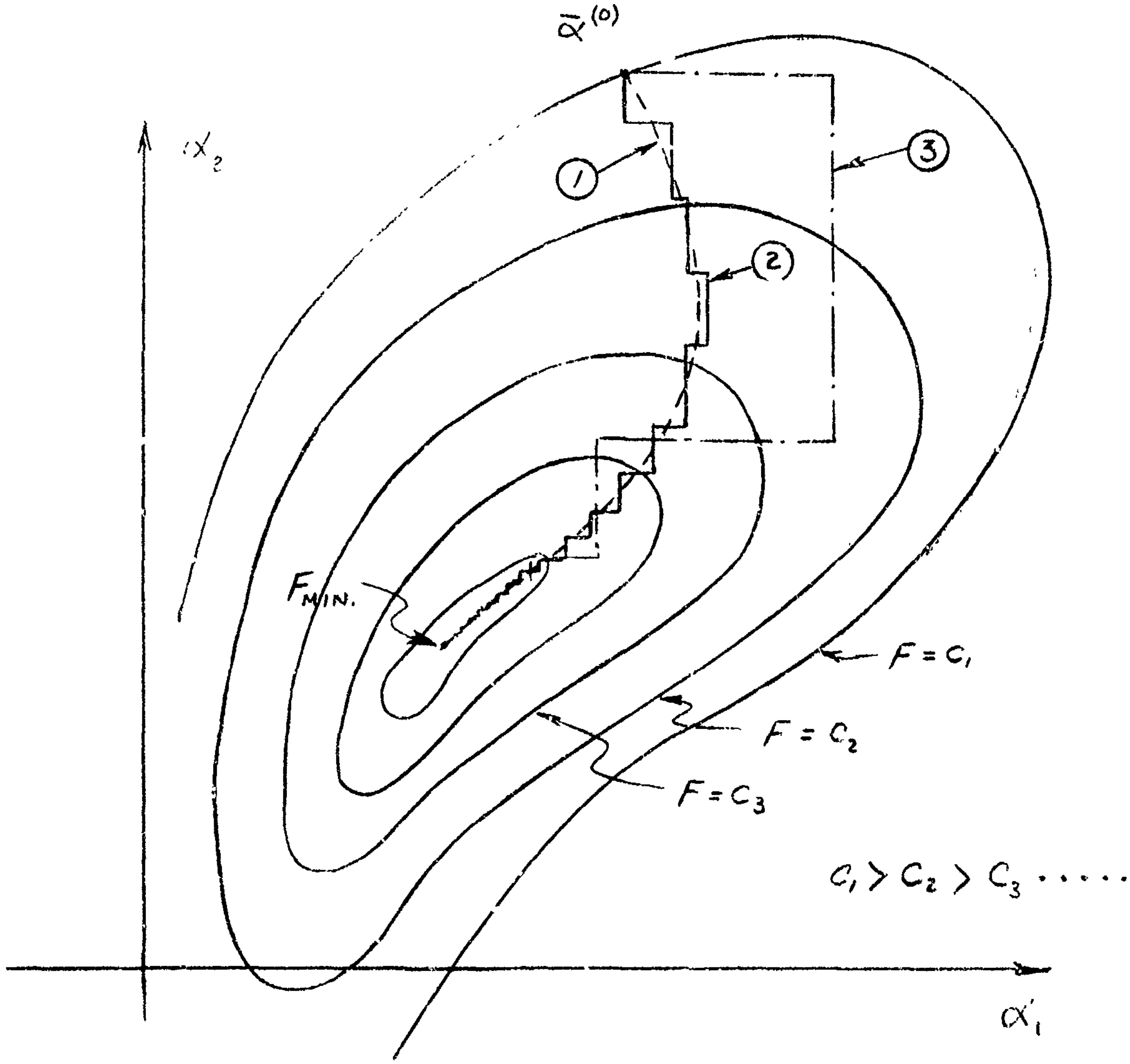


Figure 4

Contours of the Criterion Function

paths are illustrated on this Figure:

(a) Path No. 1 is a path which proceeds from the initial position along the gradient vector, i.e., in a direction always normal to the contour lines, and consequently terminates at the minimum value. This is the path usually known as the "path of steepest descent".

The adjustment strategy is based on choosing a rate of adjustment of the parameters which is proportional to the negative of the gradient vector;

$$\dot{\alpha} = -K\,\overline{\nabla F} \tag{11}$$

This equation corresponds to the two scalar equations

$$\begin{cases} \dot{\alpha}_1 = -K \dfrac{\partial F}{\partial \alpha_1} \\ \dot{\alpha}_2 = -K \dfrac{\partial F}{\partial \alpha_2} \end{cases} \tag{12}$$

Since at the minimum of the performance function the gradient approaches zero, the rate of change of the parameters likewise approaches zero and the solution becomes stationary. The steepest descent path can also be approximated by straight line segments by means of an iter[illegible] [illegible]edu[illegible] which adjusts the par[illegible] [illegible] a series of discrete steps. Thus beginning [illegible] in[illegible] val[illegible] $\overline{\alpha}^{(0)}$ evaluate the components of the gradient

$$\overline{\nabla F}\,(\alpha^{(0)}) = \left(\frac{\partial F(\overline{\alpha}^{(0)})}{\partial \alpha_1}, \frac{\partial F(\overline{\alpha}^{(0)})}{\partial \alpha_2} \right) \tag{13}$$

Using the gradient at $\alpha^{(0)}$ one can now compute a discrete parameter change vector by means of the relationship

$$(14)\quad \overline{\Delta}\alpha^{(0)} = - K\,\overline{\nabla}F(\overline{\alpha}^{(0)})$$

which is clearly a discrete form of Equation (11). A new value for the parameter vector is then defined by

$$(15)\quad \overline{\alpha}^{(1)} = \overline{\alpha}^{(0)} + \overline{\Delta\alpha}^{(0)}$$

and the process is repeated. If the steps are sufficiently small this path can be an excellent approximation to that indicated as Path No. 1, Figure 4.

(b) <u>Path No. 2</u>

The major disadvantage of the computational strategy involved in Path No. 1 is connected with the computation of the gradient vector at each point. In order to implement Equation 14 it is necessary to compute and store all n components of the gradient vector. This procedure requires either (n + 1) computer circuits operating in parallel in order to obtain all n components of the gradient vector, or n memory cells which can be used to store the components when they are computed one at a time. Methods of computation of the gradient are discussed below.

Path No. 2 is an iterative technique based on cyclical meter adjustment. Assume that the initial value of the parameters is again given by $\bar{\alpha}^{(0)}$. Now compute one component of the gradient, say

(16) $$\frac{\partial F(\bar{\alpha}^{(0)})}{\partial \alpha_1}$$

then the parameter α_1 only is adjusted to yield

(17) $$\alpha_1^{(1)} = \alpha_1^{(0)} - K \frac{\partial F(\bar{\alpha}^{(0)})}{\partial \alpha_1}$$

and the new parameter vector $\bar{\alpha}^{(1)}$ is defined as

(18) $$\begin{aligned} \bar{\alpha}^{(1)} &= (\alpha_1^{(1)}, \alpha_2^{(0)}, \alpha_3^{(0)}, \ldots \alpha_n^{(0)}) \\ &= (\alpha_1^{(1)}, \alpha_2^{(1)}, \alpha_3^{(1)}, \ldots \alpha_n^{(1)}) \end{aligned}$$

Parameter α_2 is now adjusted to yield the next point in the parameter space, etc. The process is continued until the nth parameter has been adjusted and then the cycle is repeated.

(c) Path No. 3

This path is obtained by a so-called relaxation procedure which consists of adjusting the parameters in turn such that each adjustment is continued until the performance function is minimized with respect to the particular parameter. With reference to Figure 4 if the relaxation process is begun by adjustment of parameter α_2 it can be seen that this parameter is adjusted until the criterion function reaches a minimum. At this point α_1 is adjusted until a new minimum is reached and the process is continued.

During the current study the cyclical or iterative technique of Path 2 and the relaxation method of Path 3 were instrumented by means of an analog computer using the criterion function of Equation 9. Path No. 1 was approximated by using a continuous parameter adjustment procedure based on a minimization of the time-dependent criterion function given in Equation 8. However the gradient of the latter criterion function is not strictly defined when the parameters are varied and consequently this adjustment strategy may be considered an approximation to a continuous steepest descent path, with the degree of approximation being dependent on the rate of change of the parameters. Some of the mathematical considerations involved in the formulation of the adjustment strategies and their consequent effect on the convergence and stability of the process are discussed in Sec. 5. See also Ref. 10.

3.4 The Computation of the Gradient

Two different methods were used for the computation of components of the gradient vector. The iterative strategy denoted by Path No. 2 in Figure 4 was based on computation of components of the gradient from the relation

$$(19)\quad \frac{\partial F}{\partial \alpha_i}(\bar{\alpha}^{(k)}) = \frac{F(\alpha_1^{(k)}, \alpha_2^{(k)} \ldots, \alpha_i^{(k)} + \alpha, \ldots \alpha_n^{(k)} - F(\bar{\alpha}^{(k)})}{\Delta \alpha_i}$$

where the letter k represents the computation at the kth iteration. Clearly this computation requires the availability of two mathematical models so that the differences in performance criteria from the use of the two models can be computed.

The continuous or approximate steepest descent adjustment strategy was based on the computation of components of the gradient vector by means of the method of influence coefficients (Reference 5). This method is discussed briefly in the Appendix.

4. RESULTS

4.1 General Comments

The three techniques of parameter identification discussed above have been applied to the determination of four parameters in a particular model of the human pilot given by the equation

$$(20) \qquad \ddot{z} + \alpha_1 \dot{z} + \alpha_2 z = \alpha_3 \dot{x} + \alpha_4 x$$

or in the form of a transfer function

$$(21) \qquad \frac{K(T_1 s + 1)}{(T_2 s + 1)(T_3 s + 1)}$$

where $K = \dfrac{\alpha_4}{\alpha_2}$, $T_1 = \dfrac{\alpha_3}{\alpha_4}$

and T_2 & T_3 are the roots of the quadratic

$$\left(\frac{1}{\alpha_2} s^2 + \frac{\alpha_1}{\alpha_2} s + 1\right)$$

The results obtained in this study will be evaluated with regard to

(a) feasibility of each method

(b) consistency of parameter values as obtained by the the three different methods

(c) agreement of parameter values obtained in this study with parameter values obtained in the literature

All three of the techniques employed can be considered successful in the sense that all converge to steady-state values of the four parameters in the model. The ability of the three methods to yield to convergent values of the parameters was first verified in each case by testing the method with a known transfer function, similar in form to the human operator models later employed, which was simulated on an analog computer. Following the verification of feasibility the method was then applied to actual tape recorded human pilot tracking data.

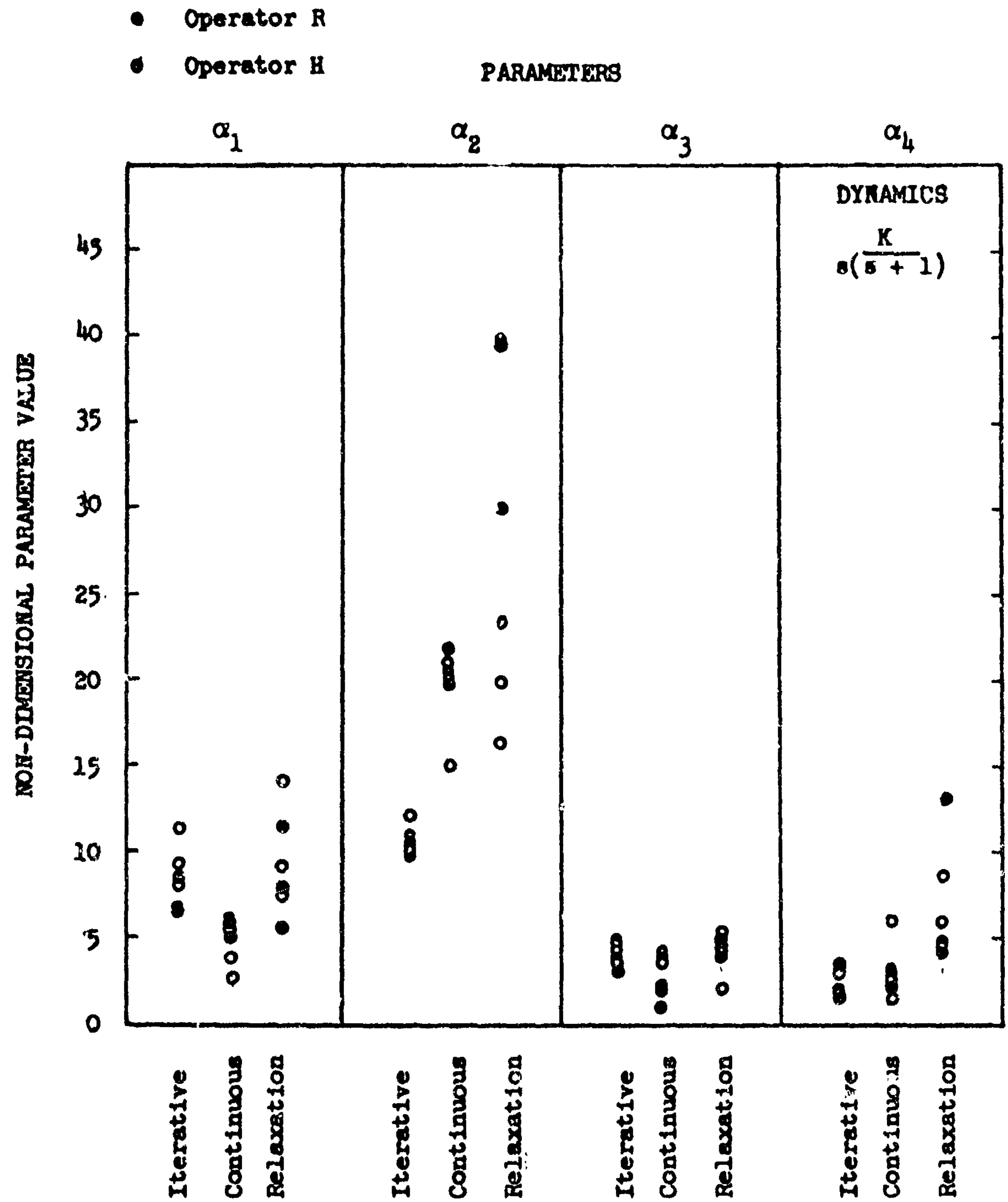
The following paragraphs present a summary of observations regarding the consistency of parameters obtained by the different techniques and a comparison of these values with previously published data. A discussion of convergence time, stability, implementation, advantages and disadvantages of the methods will be presented in Section 5.

4.2 Comparison of Results Obtained by the Three Methods

All three methods were applied to identical tracking records with the three types of controlled element dynamics mentioned above. The values of the parameters α_1, α_2, α_3, α_4, obtained from these runs are presented in graphical form in Figures 5, 6, and 7. Each of these figures presents the results obtained for one particular controlled element. The following can be noted with respect to these graphs:

(1) The values of parameters obtained by the three adjustment strategies are not equal. For parameter α_1 the continuous method yields smaller values than the other two. However this pattern is not consistent for the other parameters. Thus for parameter α_2 the iterative method yields the smallest value of the parameter. Parameter α_3 yields approximately equal values with all three methods. A slightly higher value for parameter α_4 is obtained with a relaxation method. Possible reasons for the differences are discussed in the following section, insofar as they can be shown to relate to the shape of the criterion surface.

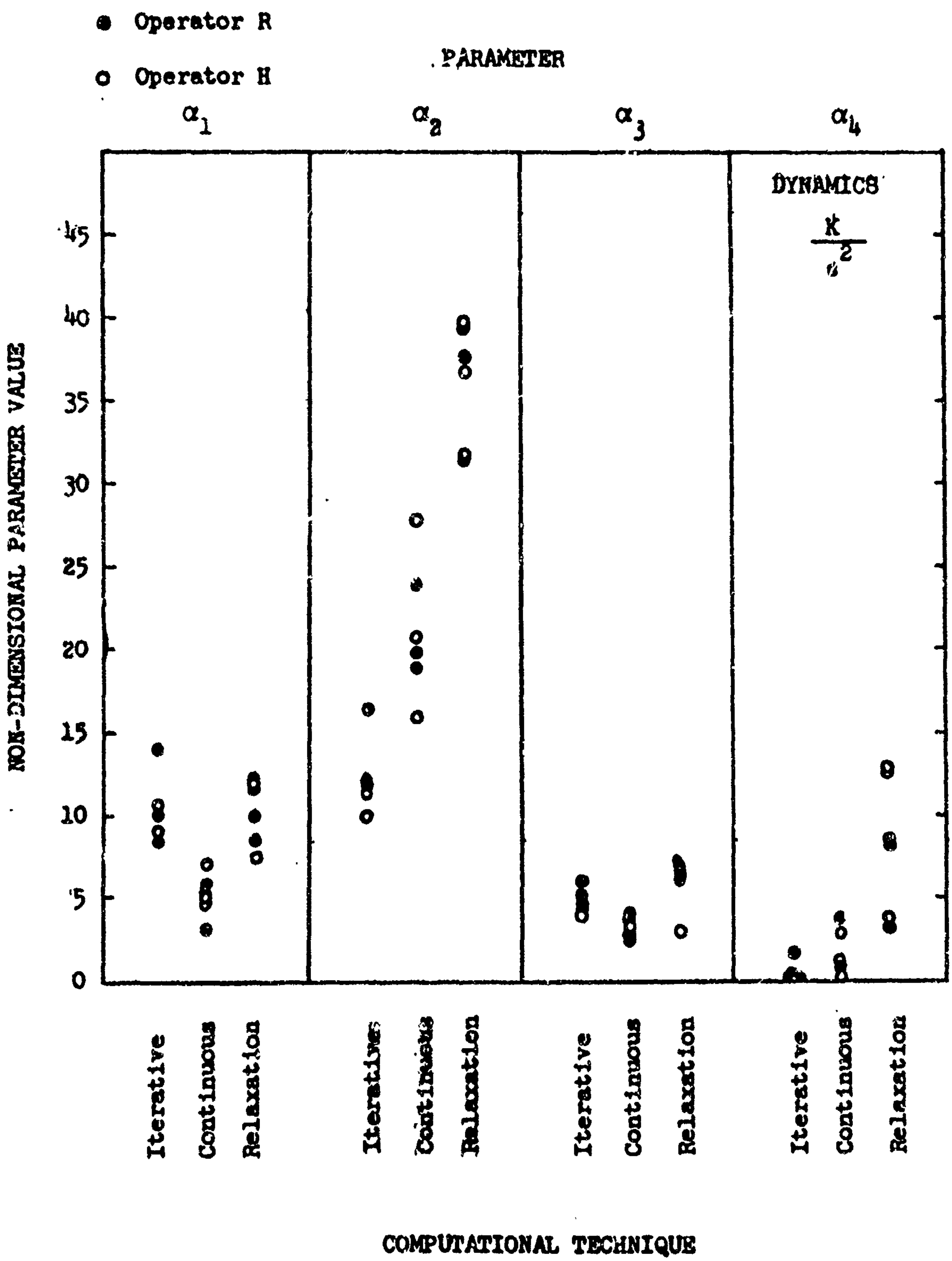
(2) The values of the parameters obtained with any one adjustment strategy vary from run to run and between operators. This variation is of the same order of magnitude as the variation between strategies.



COMPUTATIONAL TECHNIQUE

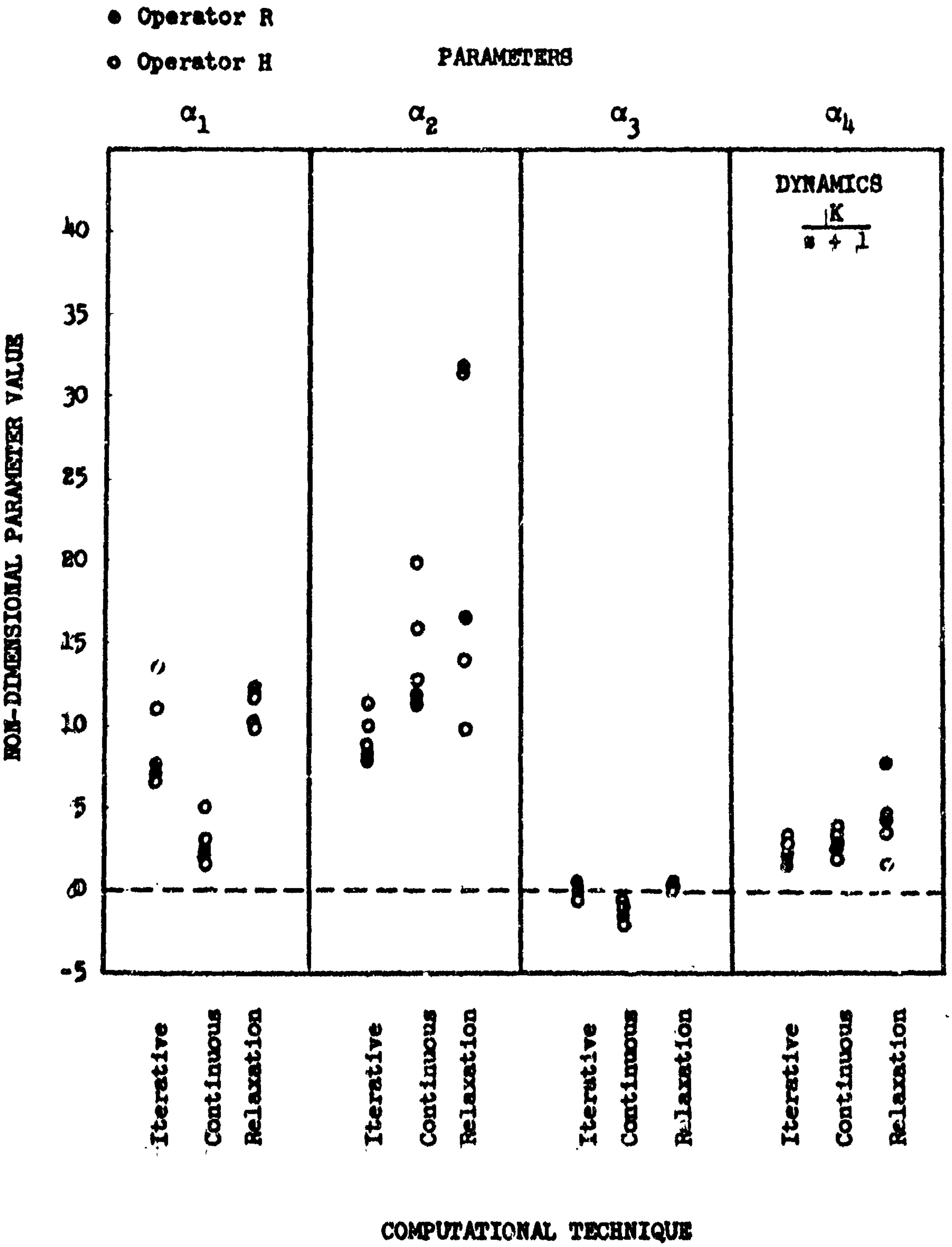
Scatter Graph of Experimentally Determined Parameter Values Using Three Adjustment Techniques

Figure 5



Scatter Graph of Experimentally Determined Parameter Values
Using Three Adjustment Techniques

Figure 6



Scatter Graph of Experimentally Determined Parameter Values Using Three Adjustment Techniques

Figure 7

(3) Insufficient data are available to determine the statistical significance of the variations.

(4) The values of parameters obtained with the two second-order dynamics are approximately equal. There is, however, an apparently significant difference between these parameters and those obtained with first order dynamics, as indicated by the difference between Figure 7 and Figures 5 and 6.

(5) While the graphs do not reveal this fact, a variation in parameter values obtained by the iterative method has been observed on successive trials with the same data. A similar variation can also be expected with the other two methods. This is explained by random signal content in the data and by corresponding variation in the signal processing sequence on the computer from run to run. Parameter values for six typical runs, averaged over the three methods, have been tabulated and are shown in Table 2, both as the values of the α's and also as values of gain and time constants in the corresponding transfer function. The following observations are made regarding these results:

(6) The majority of the data yield complex roots in the denominator of the transfer function. However, at least two of these exhibit very small imaginary parts. The significance of these complex roots is discussed in the following section.

(7) The results from any two runs with the same dynamics are quite consistent. Greater consistency is found in the values of gain K obtained from any two runs than in the values of the time constants.

TABLE 2

AVERAGED PARAMETER VALUES OBTAINED FOR THREE CONTROLLED ELEMENT DYNAMICS

Dynamics	Run No.	Coefficients α_1	α_2	α_3	α_4	Elements of Transfer Function K	T_1	T_2	T_3
$\frac{12.5}{S(S+1)}$	1	6.3	19.0	3.67	5.8	.30	.63	.17+.15j	.17-.15j
	5	6.8	14	4.15	3.7	.26	1.1	.24+.11j	.24-.11j
$\frac{K}{S^2}$	7	8.3	23	3.4	2.3	.10	1.48	.18+.1j	.18-.1j
	12	9.5	23	5.5	1.8	.078	3.0	.21+.03j	.21-.03j
$\frac{K}{S+1}$	15	8	13.2	-.73	3.4	.26	-.21	.18	.42
	17	6.6	12.2	.067	3	.25	.022	.27+.082j	.27-.082j

4.3 Comparison with Previously Published Data

A comparison of the results obtained in this study with those previously published by Adams (Reference 9) is indicated in Table 3. It can be noted from an examination of this table that the results are of the same order of magnitude. In the absence of statistically significant samples, a closer comparison cannot be made at this time. The following points should be noted in connection with the comparison of Table 3:

(a) The excitation signal break frequency was .1 rad/sec in both studies. However, the STL study used a third-order filter and the NASA study presumably used a first-order filter.

(b) The gain term in the definition of the dynamics cannot be compared more precisely without additional data. The STL gain is a loop gain including the oscilloscope gain in volt/cm, thus yielding units of cm^{-1}. The

TABLE 3

COMPARISON OF EXPERIMENTAL RESULTS WITH PUBLISHED RESULTS

Controlled Dynamics (CM^{-1})	Disturbance Break Frequency (rad/sec)	STL RESULTS (Operator H)				Controlled Dynamics (IN^{-1})	Published Results (Adams, Ref. 9, Pilots F ¢ H)		
		K	T_1	T_2	T_3		K	T_1	$T_2=T_3$
$\frac{12.5}{s(s+1)}$	1	.26	1.1	2.4+.11j	2.4-.11j	$\frac{10}{s(s+1)}$	.25	2.0	.33
$\frac{12.5}{s(s+1)}$	1	.30	.63	.17+.15j	.17-.15	$\frac{10}{s(s+1)}$	.77	.85	.15
$\frac{12.5}{s^2}$	1	.078	3.0	.21+.03j	.21-.03j	$\frac{10}{s^2}$	.167	1.78	.22
$\frac{12.5}{s^2}$	1	.10	1.48	.18+.1j	.18-.1j	$\frac{10}{s^2}$	.62	1.0	.125

NASA gain is based on the controlled element dynamics only, thus yielding units of volts $^{-1}$. An exact comparison is possible only if it is assumed that an oscilloscope gain of 1.25 volts/cm was used.

4.4 Time Recordings

The actual performance of each of the three methods can be examined more directly from strip chart recordings obtained during typical runs. The recordings contain the time history of the four parameters as well as inputs and outputs of model and human operator and the criterion function. The symbols used on the figures can be identified by reference to the following block diagram.

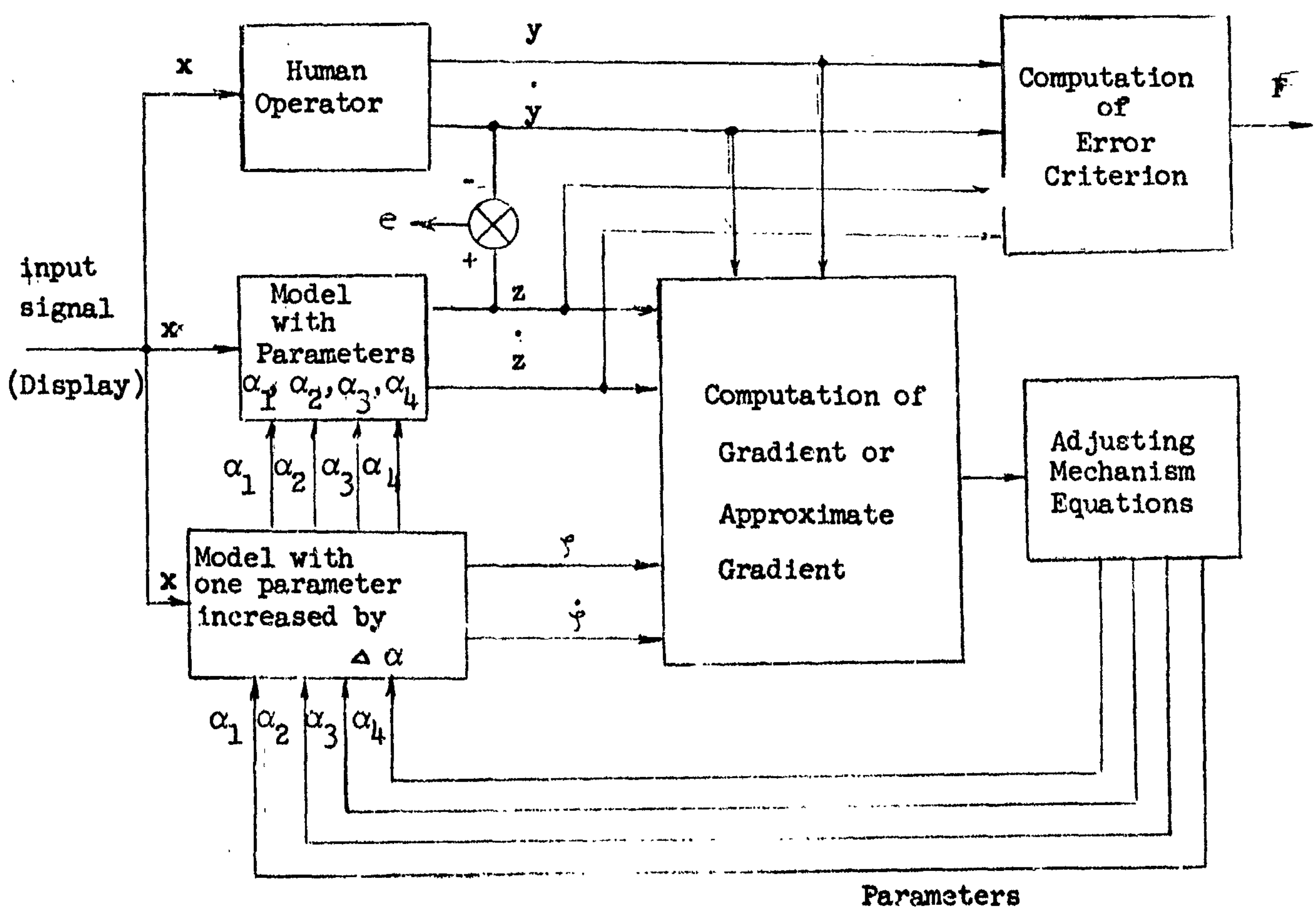
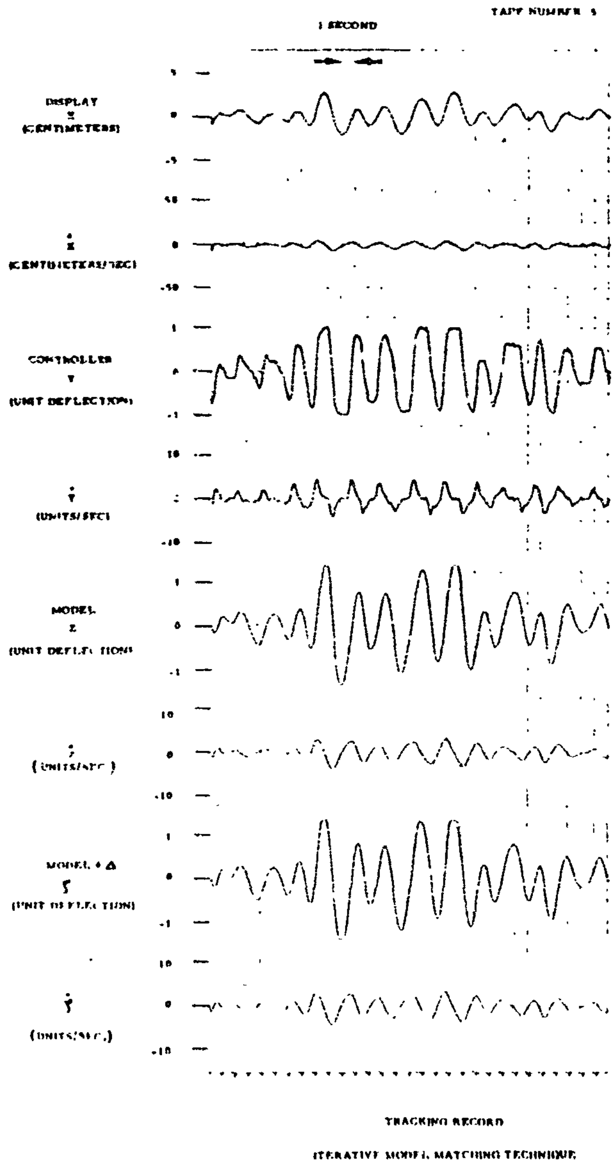


Figure 8
Block Diagram of Model Matching Technique

(a) Iterative Method - Figures 9 - 13 present typical results, as follows:

Figure No.	Quantities Recorded	Remarks
9	$x, \dot{x}$ $y, \dot{y}$ $z, \dot{z}$ $\xi, \dot{\xi}$	1. Shows traces after completion of parameter adjustment. 2. Controller motion was limited to ± 1, Model output was larger at some points
10	α_i $\frac{\partial F}{\partial \alpha_i}$ F	1. α_1 and α_2 show slight overshoot. 2. α_3 and α_4 show oscillatory behavior.
11	α_i $\frac{\partial F}{\partial \alpha_i}$ F	1. Marked overshoot of α_3 2. Convergence to steady-state. Note behavior of error criterion F.
12	α_i $\frac{\partial F}{\partial \alpha_i}$ F	1. Convergence to steady-state without overshoot in any parameter
13	α_i $\frac{\partial F}{\partial \alpha_i}$ F	1. Note marked effect of α_3 on error criterion. 2. Slight overshoot in α_1



TRACKING RECORD

ITERATIVE MODEL MATCHING TECHNIQUE

Figure 9

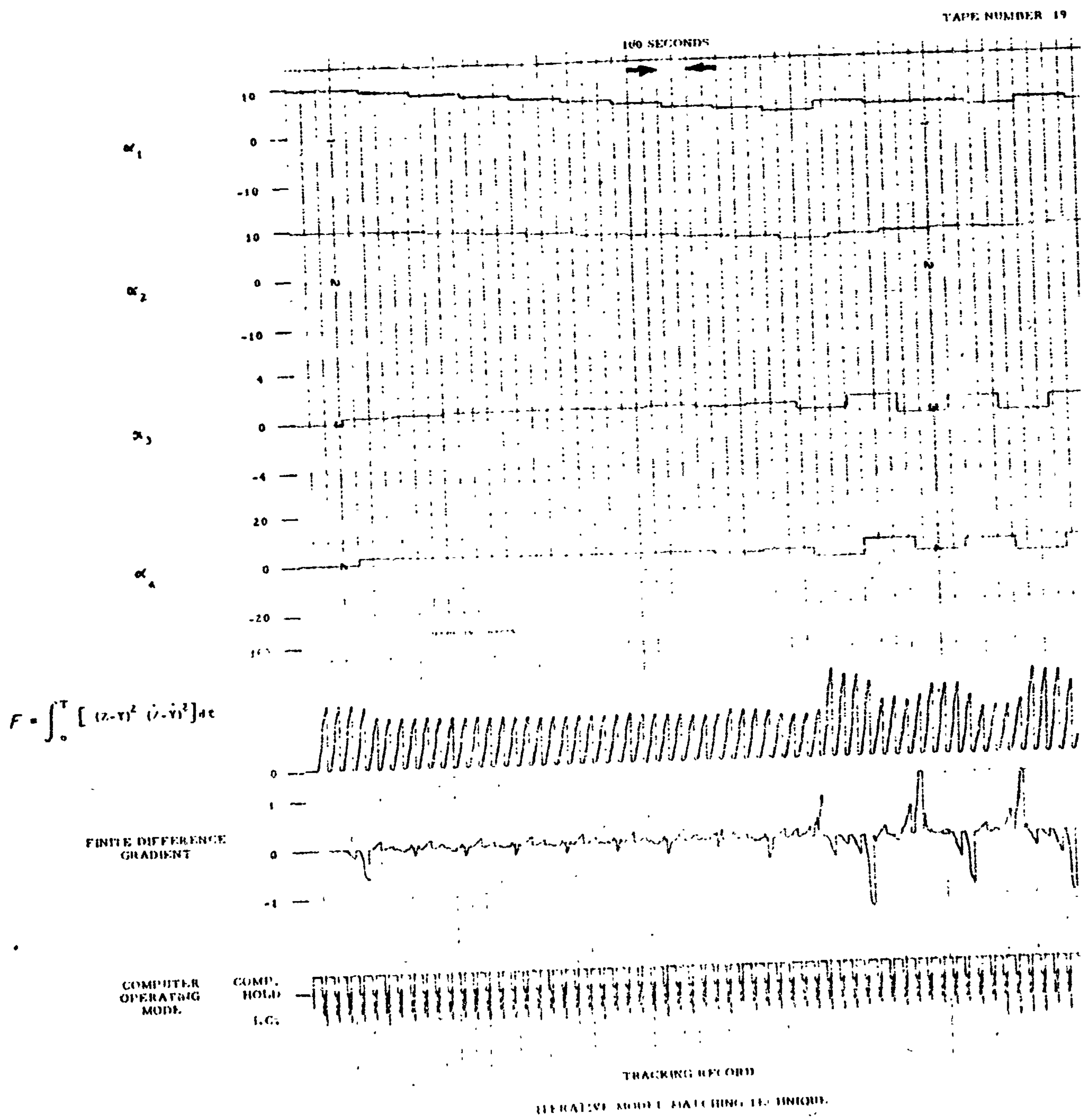
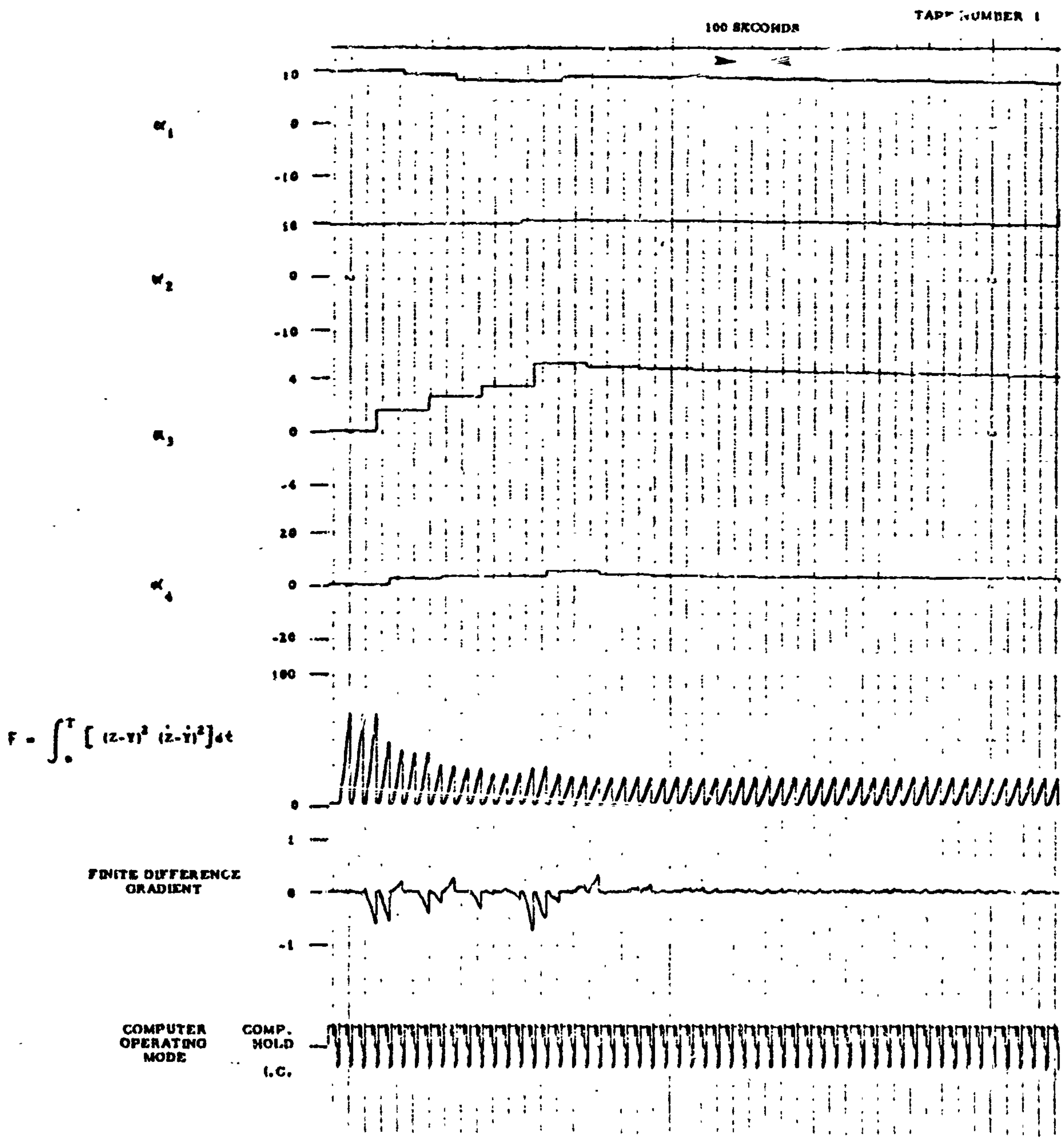

TAPE NUMBER 19
100 SECONDS
10
0
-10
10
0
-10
4
0
-4
20
0
-20
$F = \int_0^T [(Z-Y)^2 \; (\dot{Z}-\dot{Y})^2] dt$
0
1
FINITE DIFFERENCE GRADIENT
0
-1
COMPUTER OPERATING MODE
COMP.
HOLD
I.C.
TRACKING RECORD
ITERATIVE MODEL MATCHING TECHNIQUE

Figure 10



TRACKING RECORD

ITERATIVE MODEL MATCHING TECHNIQUE

Figure 11

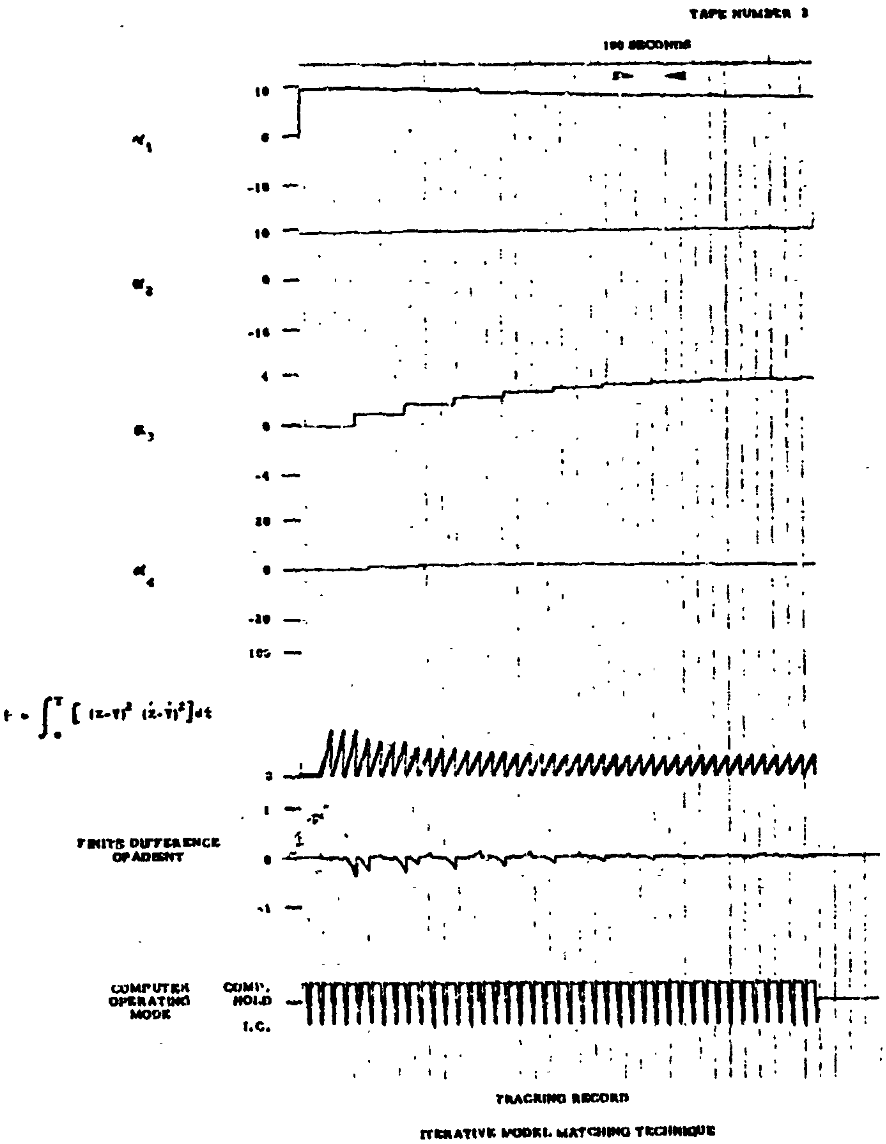
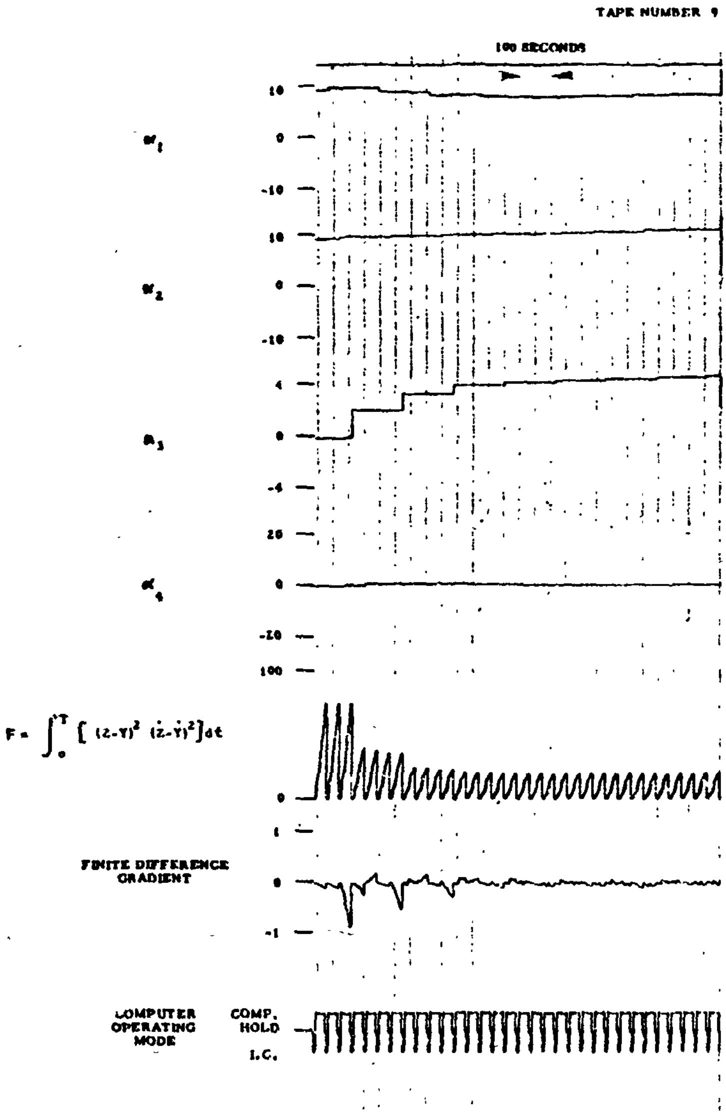

TAPE NUMBER 3
100 SECONDS
TRACKING RECORD
ITERATIVE MODEL MATCHING TECHNIQUE
FINITE DIFFERENCE GRADIENT
COMPUTER OPERATING MODE
COMP.
HOLD
I.C.

Figure 12



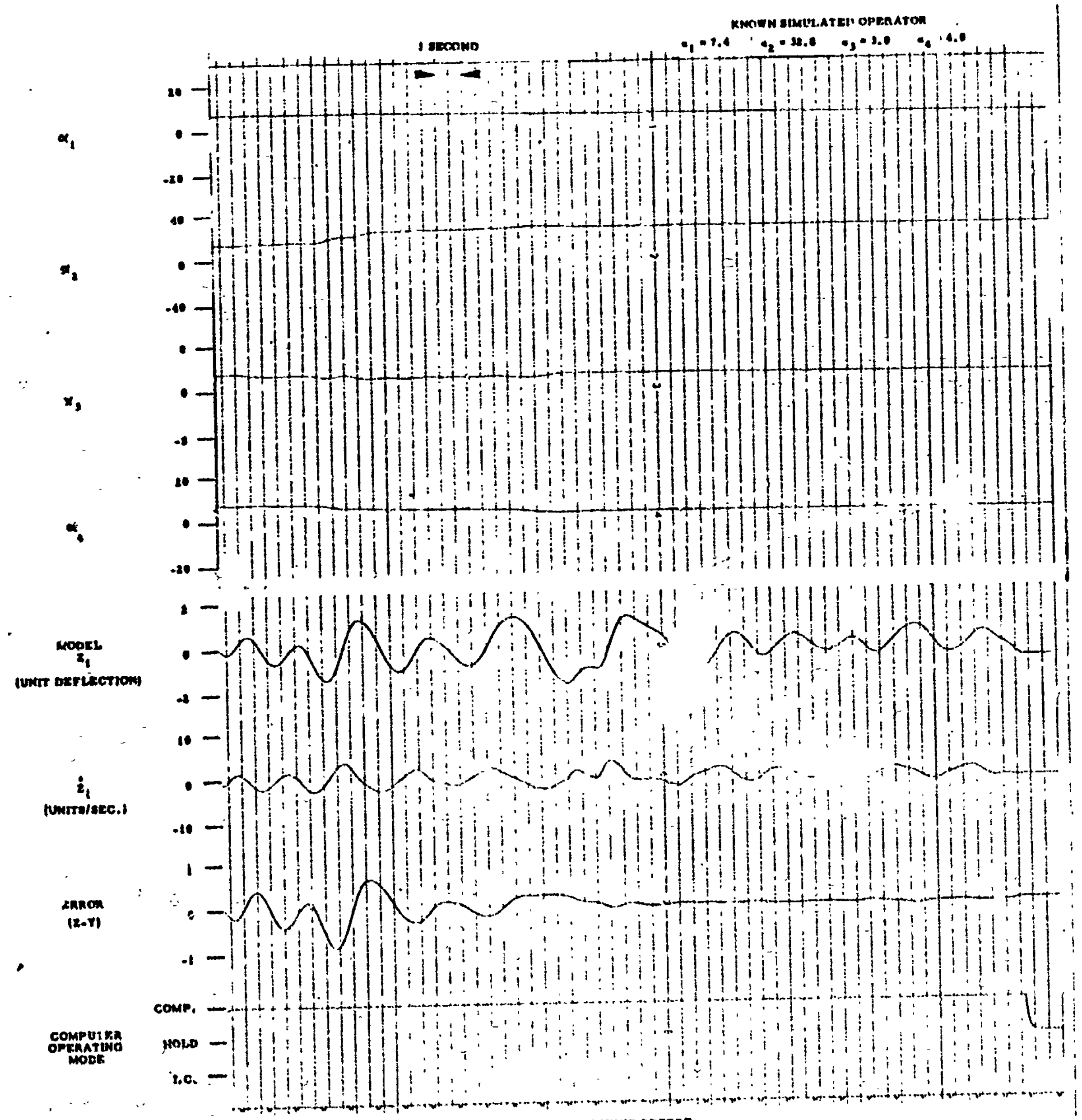
TRACKING RECORD

ITERATIVE MODEL MATCHING TECHNIQUE

Figure 13

(b) Continuous Method - Figures 14 - 17 present typical results as follows:

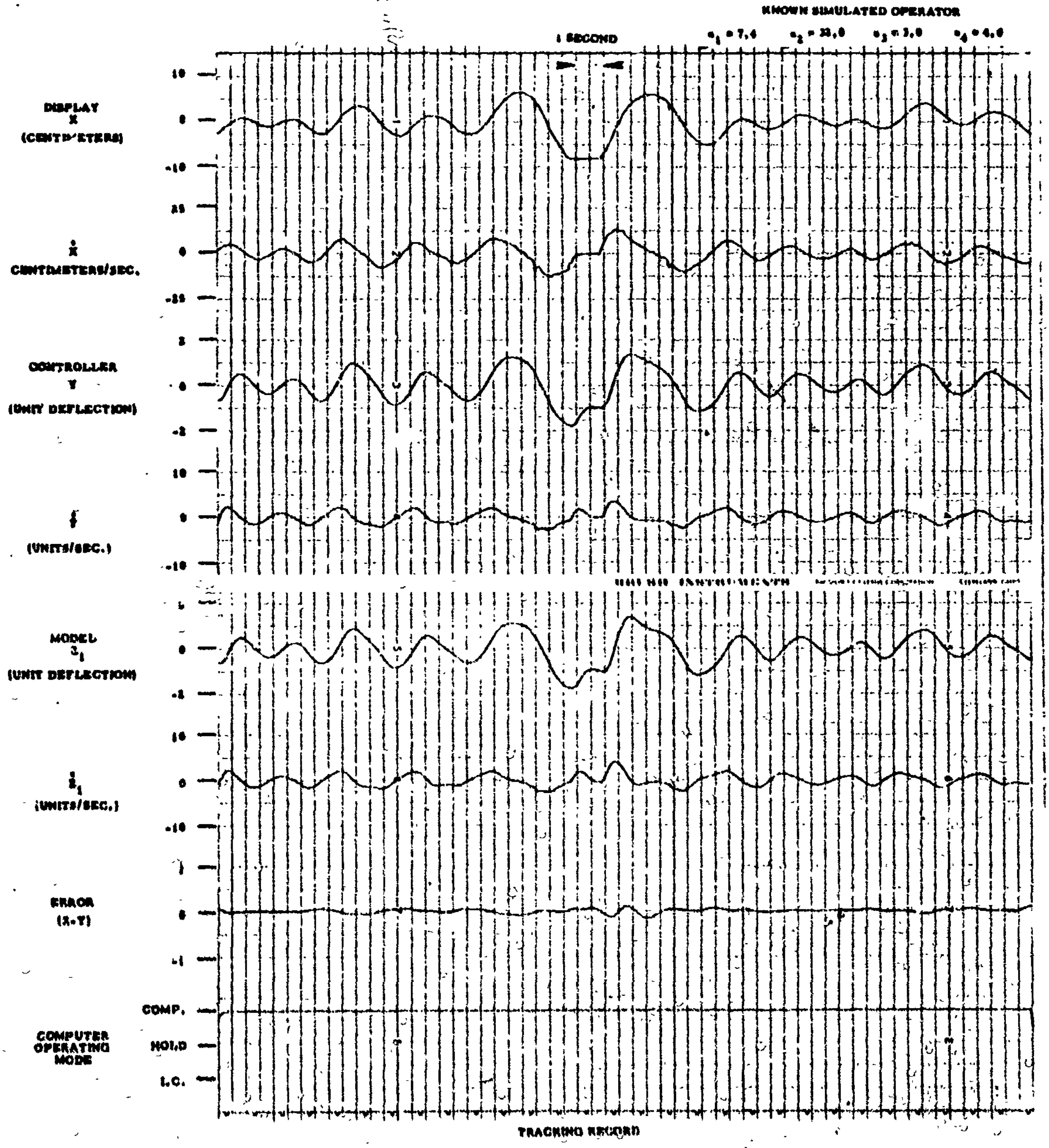
Figure No.	Quantities Recorded	Remarks
14	α_i $z, \dot{z}$ e	1. Coefficients of known transfer function ("analog pilot") are matched 2. Error e is reduced from a maximum of .65 to less than .1 in 15 secs
15	$x, \dot{x}$ $y, \dot{y}$ $z, \dot{z}$ e	1. Same known transfer function as Figure 15, after completion of parameter adjustment. 2. Shows excellent match of model and simulated pilot
16	$x, \dot{x}$ $y, \dot{y}$ $z, \dot{z}$ e	1. Human operator tracking. Traces shown after reaching steady state. 2. Note the relatively large magnitude of the error in spite of similarity of human and model output.
17	α_i z e	1. Parameters during same period as Figure 16. Changes in parameters reveal that operator was changing his performance characteristics.



TRACKING RECORD

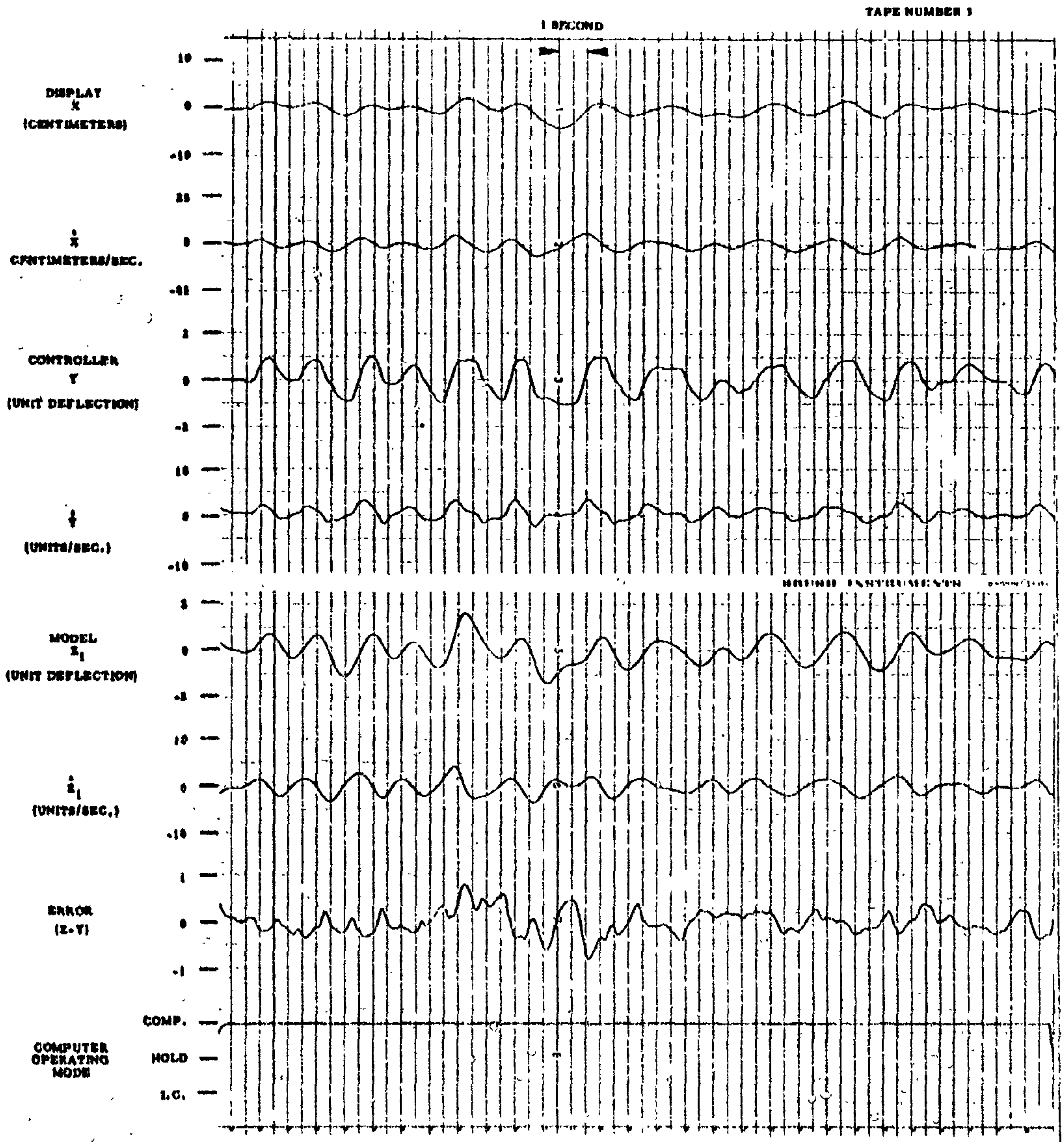
CONTINUOUS MODEL MATCHING TECHNIQUE

Figure 14



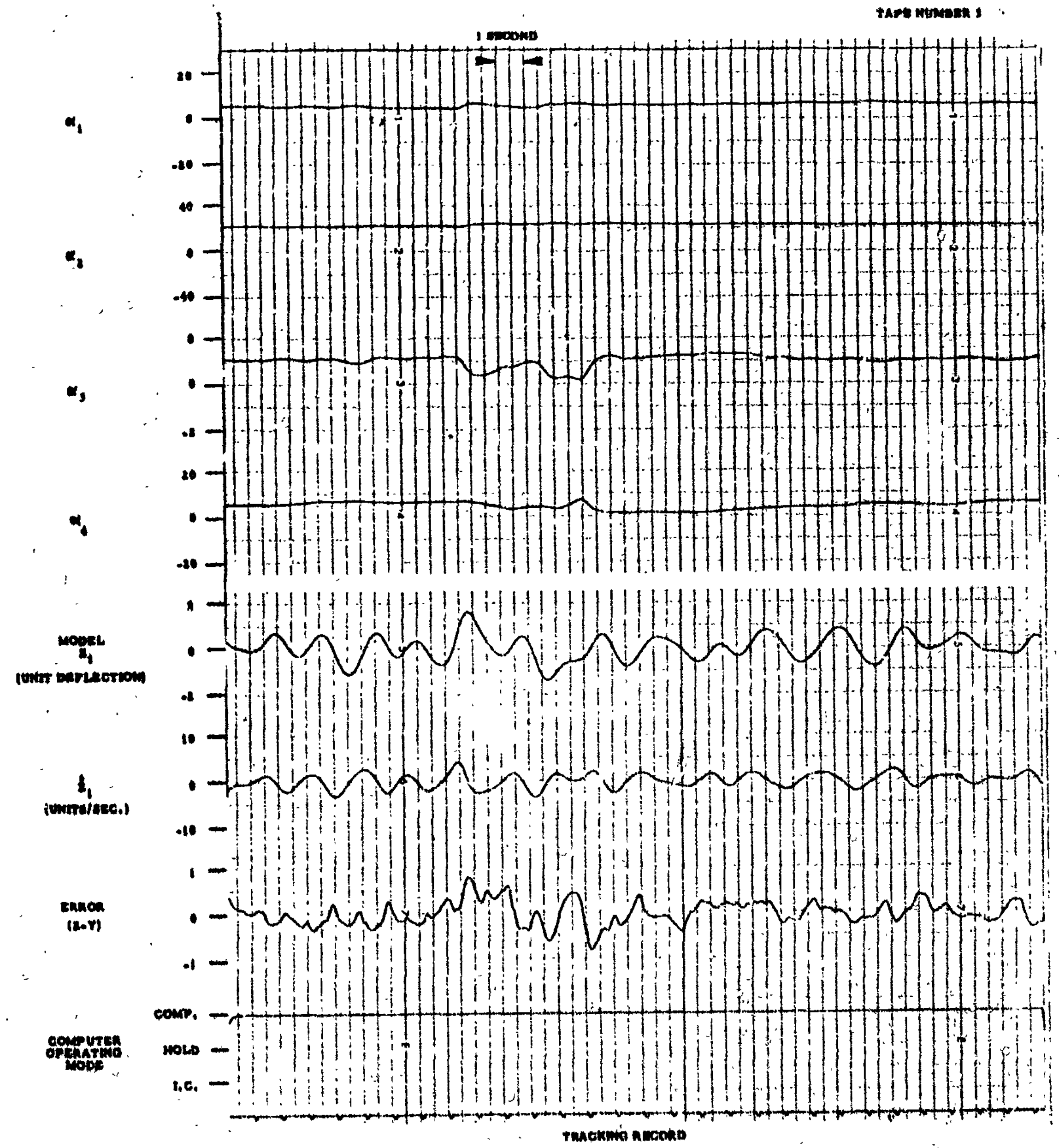
TRACKING RECORD
CONTINUOUS MODEL MATCHING TECHNIQUE

Figure 15



TRACKING RECORD
CONTINUOUS MODEL MATCHING TECHNIQUE

Figure 16



TRACKING RECORD
CONTINUOUS MODEL MATCHING TECHNIQUE

Figure 17

(c) Relaxation Method - Figures 18 - 22 show the time traces of x, $\dot{x}$, y, $\dot{y}$, z, $\dot{z}$, and the error criterion F after various stages of adjustment.

Figure No.	Remarks
18	Initial values of parameters. Since $\alpha_3 = \alpha_4 = 0$, the model output is zero.
19	New run after finding optimum value for α_4. α_1, α_2, and α_3 still have their initial value. Note small model output and slower increase in F.
20	New run after finding optimum value for α_3. α_1, α_2 and α_4 as in Figure 19. Note improvement in z and F.
21	New run after finding optimum value for α_2. α_1, α_3, α_4 as in Figure 20. Note the very slight effect of this parameter.
22	New run after finding optimum value for α_1. α_2, α_3 and α_4 as in Figure 21. Note marked improvement in F and close resemblance of z and y.

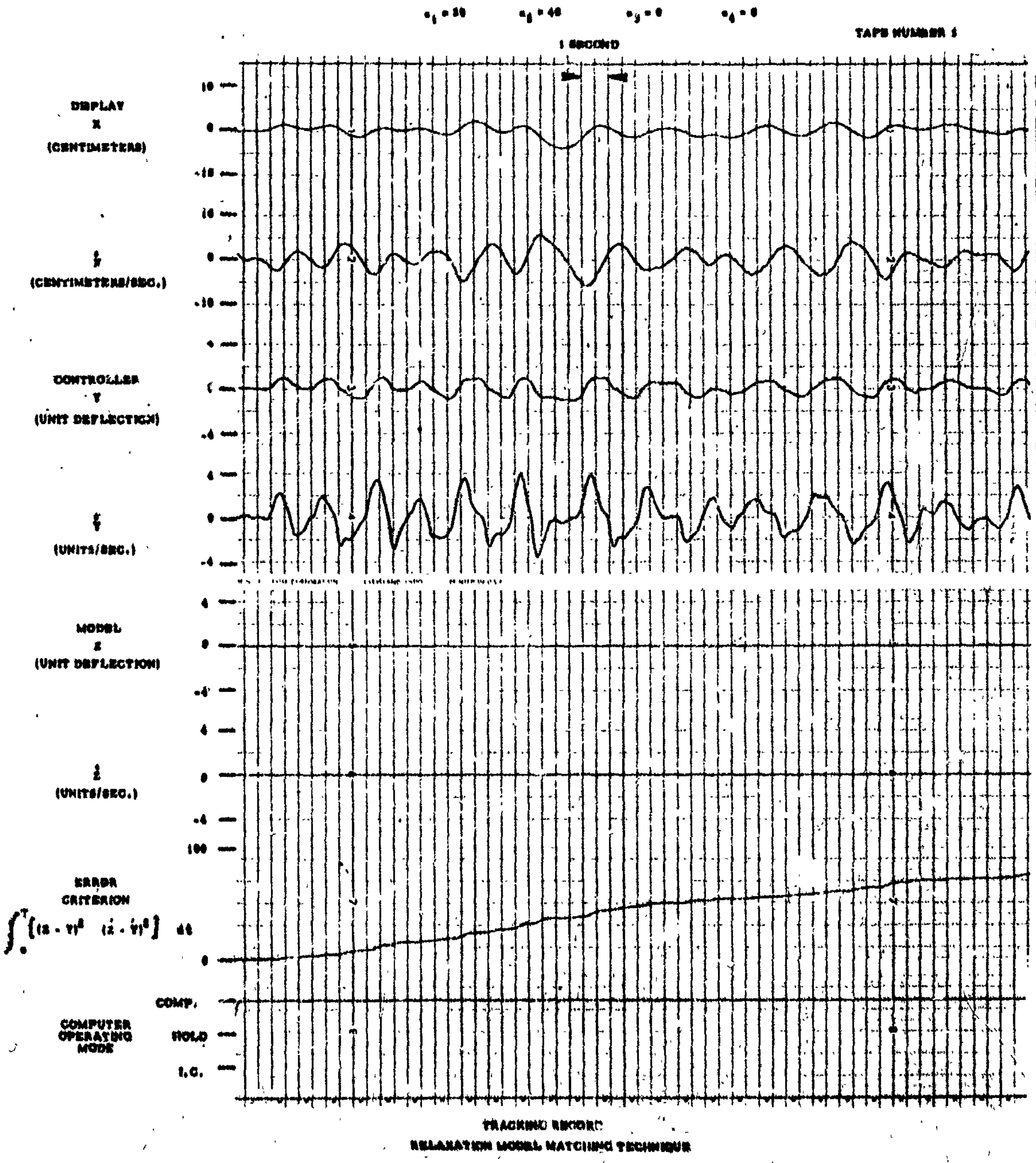
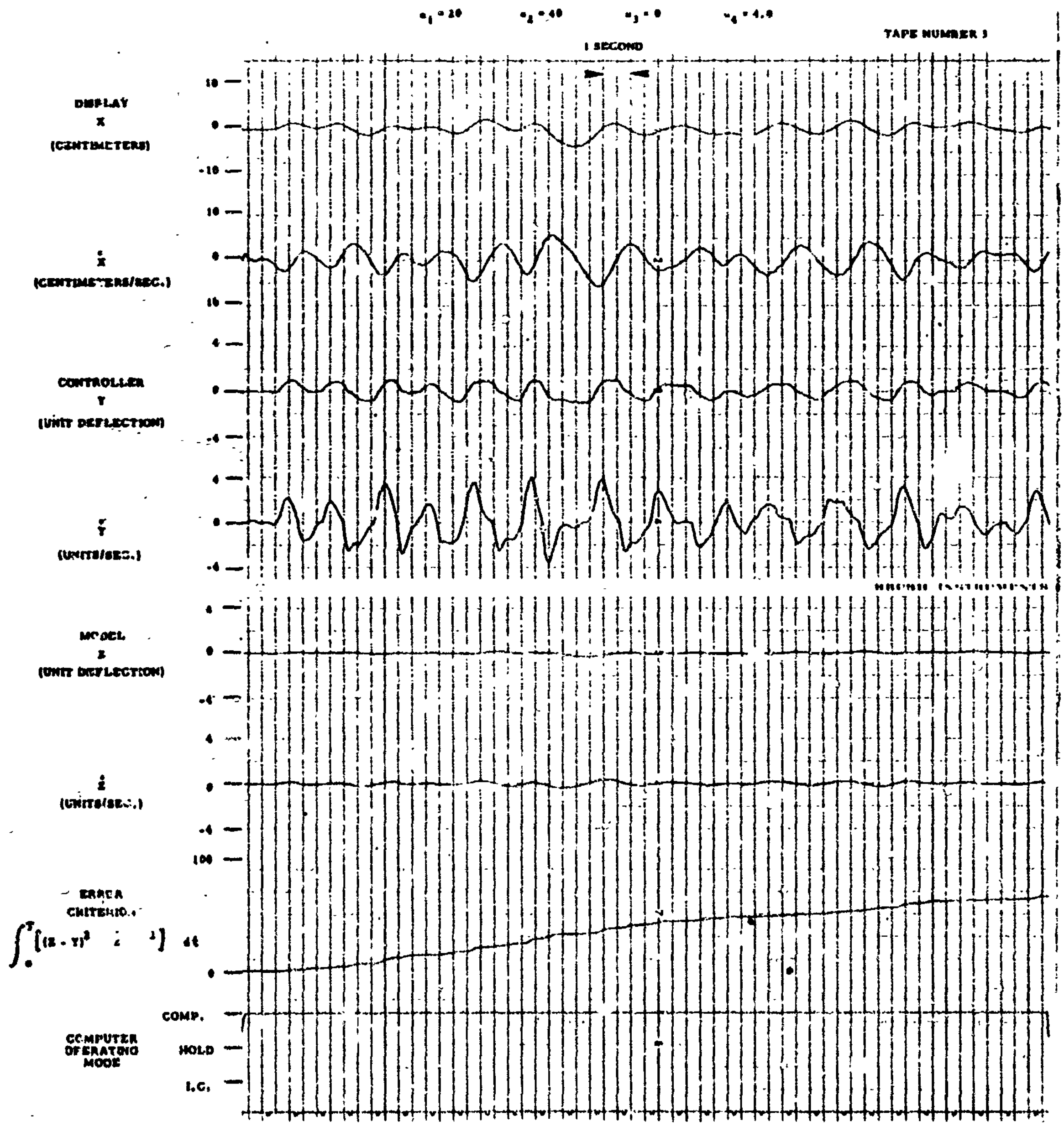
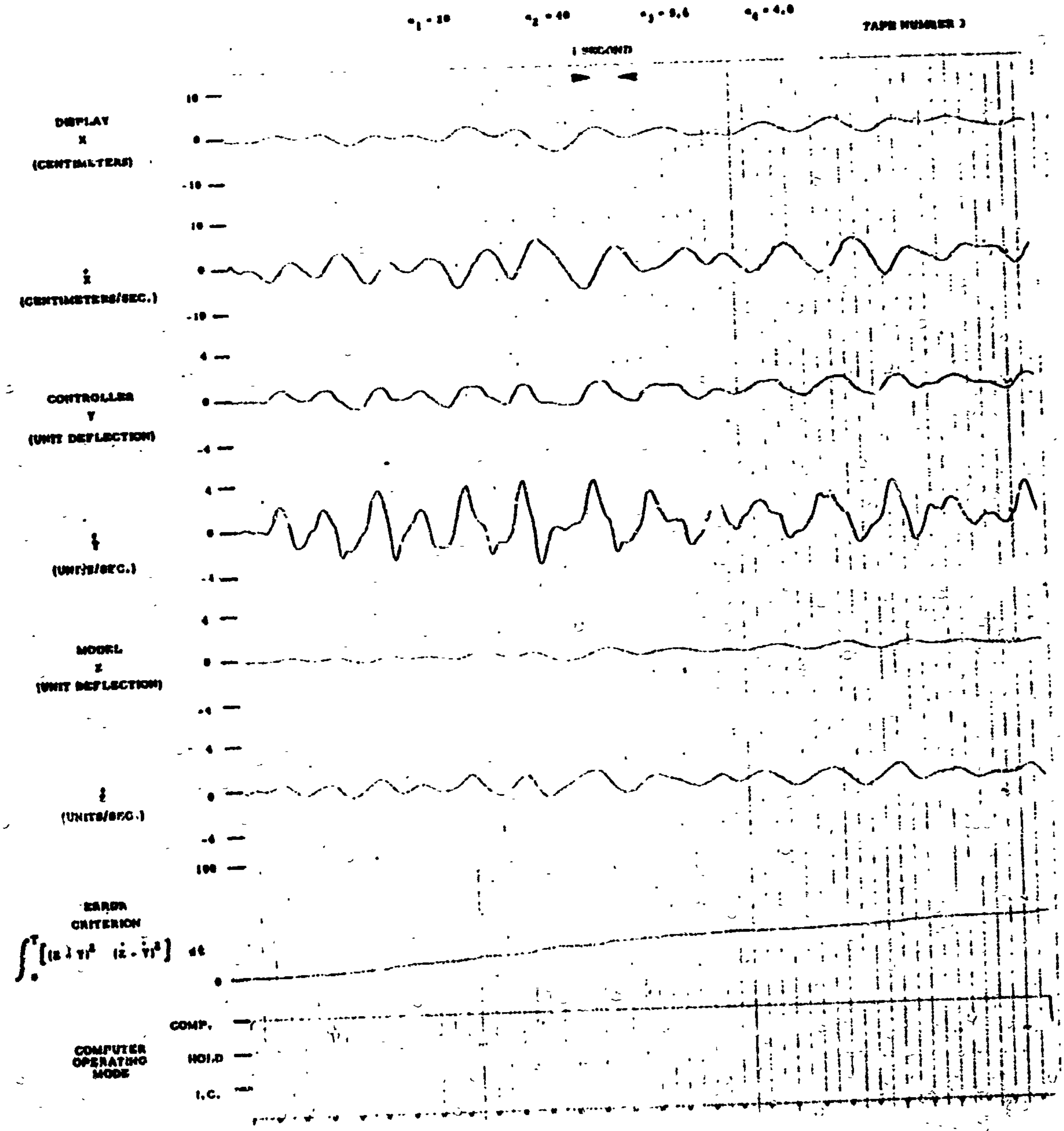


Figure 18



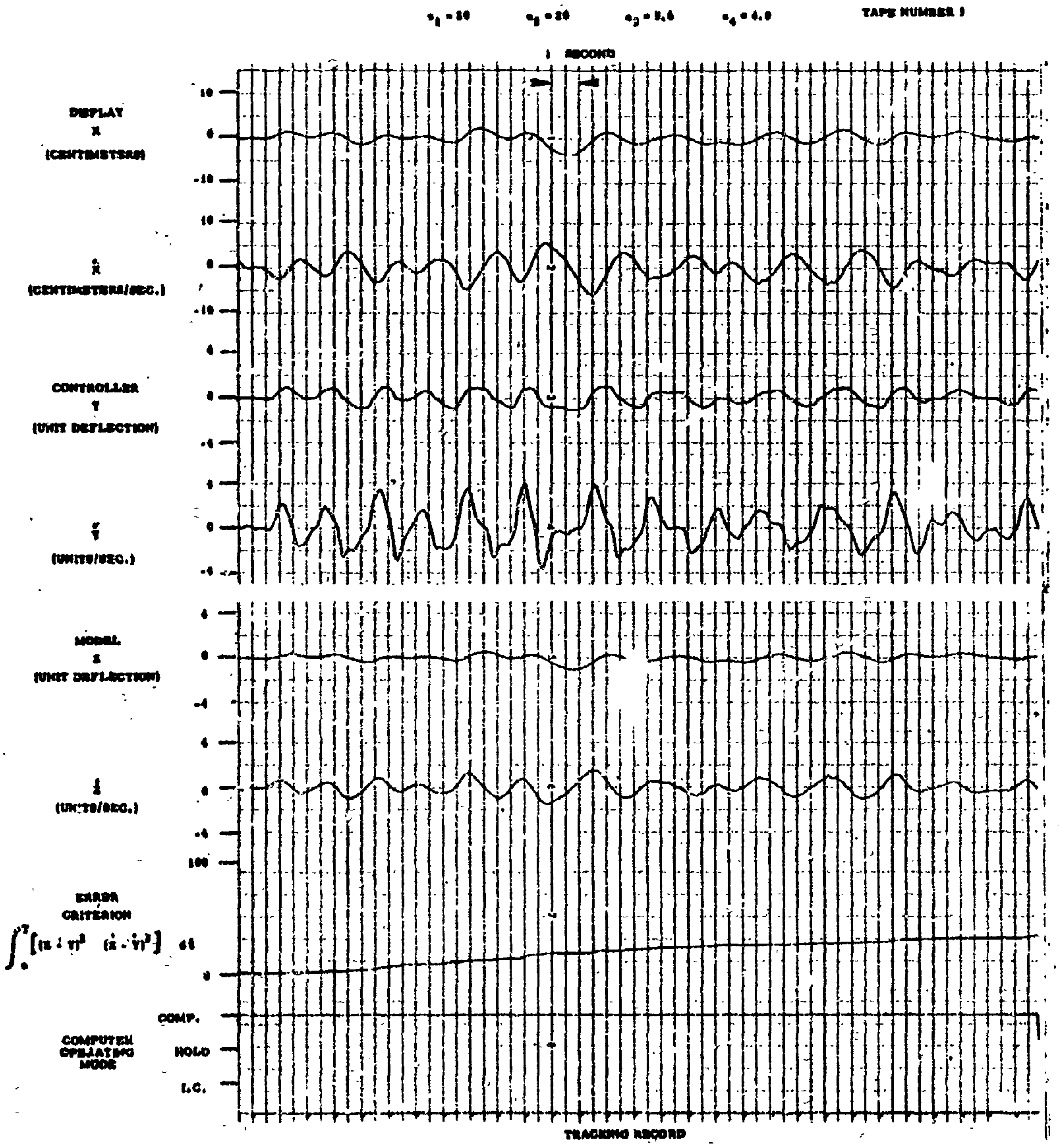
TRACKING RECORD
RELAXATION MODEL MATCHING TECHNIQUE

Figure 19



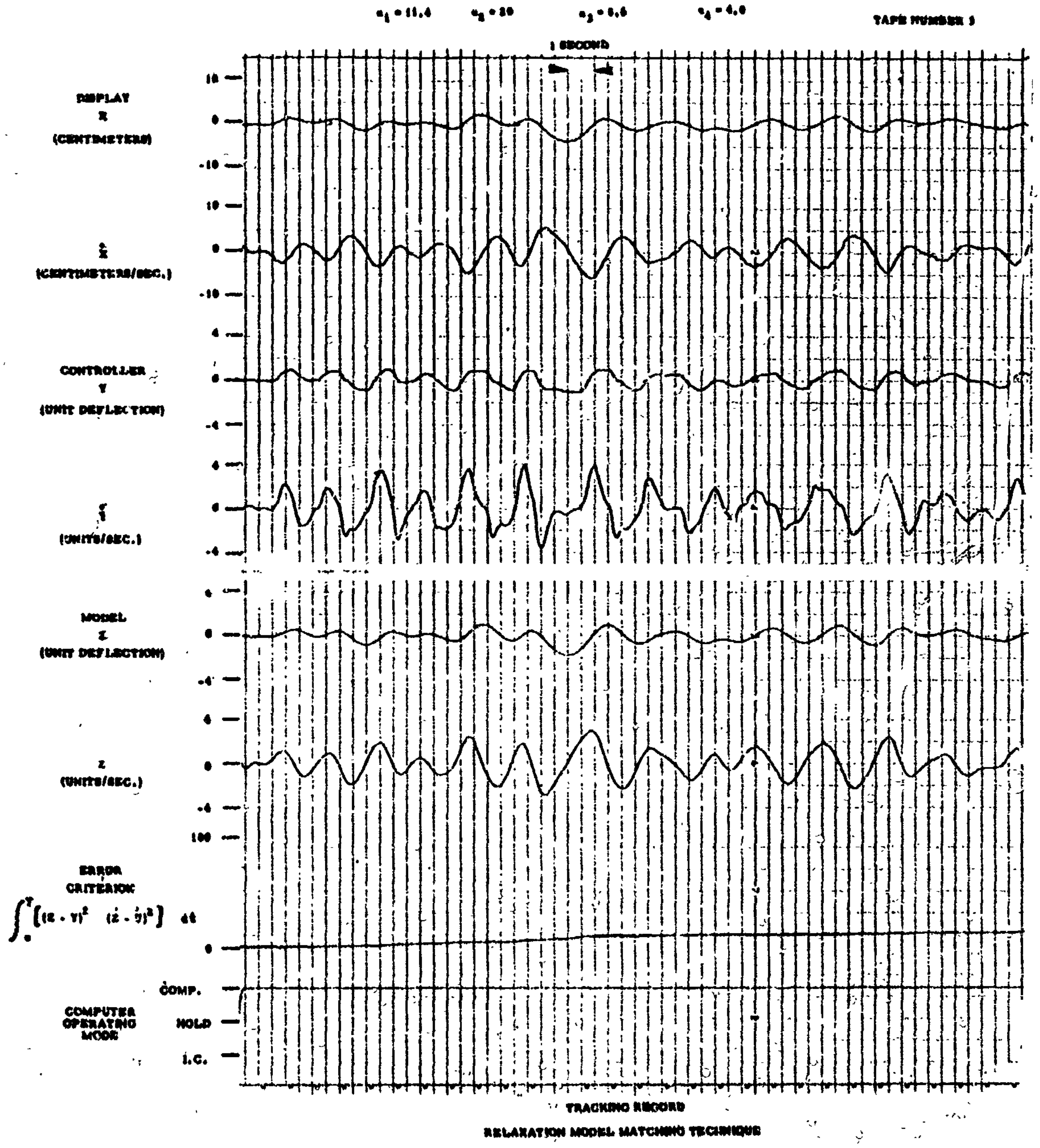
TRACKING RECORD
RELAXATION MODEL MATCHING TECHNIQUE

Figure 20



TRACKING RECORD
RELAXATION MODEL MATCHING TECHNIQUE

Figure 21



a_1 = 11.4
a_2 = 20
a_4 = 4.0
1 SECOND
DISPLAY
(CENTIMETERS)
(CENTIMETERS/SEC.)
CONTROLLER
(UNIT DEFLECTION)
(UNITS/SEC.)
MODEL
(UNIT DEFLECTION)
(UNITS/SEC.)
ERROR
CRITERION
COMPUTER
OPERATING
MODE
COMP.
HOLD
I.C.
TRACKING RECORD
RELAXATION MODEL MATCHING TECHNIQUE

Figure 22

5. DISCUSSION

A preliminary assessment of the relative advantages of the different adjustment methods can be made in terms of computer implementation, convergence and stability, mathematical consideration, and suitability for "on-line" applications.

5.1 Computer Implementation

A comparison of the continuous and step-by-step adjustment techniques indicates a substantial difference in complexity of computer programming and operation. With the continuous technique, very little programming of computer logic and mode switching is required, and repetitive operation using recorded input and output signals is avoided. Many analog computer facilities are equipped primarily for continuous operation, and such operations may be preferred. On the other hand, the continuous method requires a large number of multipliers, which are avoided with the iterative technique. The iterative technique requires logical circuits and analog memory circuits for storage of parameter values.

The relaxation method, as used in this study, was based on manual adjustment of parameters which had to be resorted to for lack of sufficient computer capacity. Consequently, it is difficult to compare the implementation difficulties of this method with other methods. However, the relaxation method is basically a form of iteration, and thus requires iterative and storage circuits.

5.2 Validity of the Results

The results show that all three methods considered yield parameter values which are approximately equal. However, the final values of the performance criterion function (in the iterative and relaxation methods) are seldom smaller than 30 percent of the values obtained with initial arbitrary parameter values. Furthermore, while the model output, after parameter adjustment, does resemble the human pilot's output, this resemblance is not sufficiently close to make the criterion function approach zero. The differences could be due to:

(a) Omission of the time-delay (reaction-time) term known to exist in the human

(b) Inadequate complexity of the model

(c) Inadequate training and, hence, lack of consistency on the part of the operators.

Not enough data are available to distinguish among these possible causes at the present time.

5.3 Convergence and Stability

This is one of the most serious problems in the model-matching technique, especially since convergence is difficult to define for the type of problem being studied. The human pilot models are based on an assumption and, consequently, the "true" or "correct" values of the parameters are not known. Therefore, one cannot define convergence with reference to correct values of the parameters. As an alternative to a quantitative definition of convergence, it is possible to examine the parameter values to determine whether they: (a) approach a steady-state value, (b) do not approach a steady-state value, or (c) oscillate about some value. All three methods yielded parameter values which either approached or oscillated about a steady-state value.

(a) Continuous Method.

Oscillations in the parameter values tend to occur when the gain K in the adjusting loop is chosen too large. To avoid such oscillations, the gain was kept sufficiently low. The continuous adjustment method exhibits this tendency to oscillation because of the feedback path present in the adjusting mechanism. Analytical stability criteria are lacking at the present time. The continuous method may require the design of complex phase compensation networks to insure stability during parameter adjustment.

(b) Iterative Method.

The approach to the steady-state with the iterative method may exhibit overshoot phenomena and oscillations, as seen in several records of solutions obtained by the step-by-step adjustment (Figures 9 and 10). The oscillation is probably due to one or both of the following causes: (1) excessive step size in the vicinity of the minimum of the criterion function and (2) the method used to define the sensitivity functions $\partial F/\partial \alpha j$. It is clear that an excessively large step can result in oscillation. The second item, however, is not quite as obvious. The sensitivity function was computed from the relationship

$$(22) \quad \frac{\partial F}{\partial \alpha_j} \doteq \frac{F(\alpha_1,\alpha_2,\ldots\alpha_j+\Delta\alpha_j,\alpha_n) - F(\alpha_1,\alpha_2,\ldots\alpha_j,\ldots\alpha_n}{\Delta\alpha_j} \qquad j=1,2,\ldots n$$

The validity of this approximation to the slope of the profile of the criterion function with respect to α_j improves as $\Delta\alpha \rightarrow 0$. However, the analog computer implementation becomes worse as $\Delta\alpha$ approaches zero due to computer noise. It is also possible to make use of the influence coefficient technique for computing the sensitivity functions exactly, but that was not done for the iterative method.

The choice of step size is in direct analogy to the choice of the continuous adjustment gain factor K in equations (11) and (14). Care must be taken in either case to limit the adjustment gain to values permitting rapid convergence while avoiding excessive hunting near the steady-state values.

(c) Relaxation Method

The relaxation method was based on finding the minimum of the criterion function (on a digital voltmeter) and manual adjustment of the parameters. No attempt was made to improve the resolution of the voltmeter near the minimum, thus resulting in somewhat uncertain convergence to the final parameter values.

5.4 Dependence of Parameter Values on the Shape of the Criterion Surface

The ability of the automatic model-matching techniques (both continuous and iterative) to produce a well-defined steady-state value for the parameters depends to some extent on the shape of the criterion surface in the parameter space.

An inspection of the characteristics of the criterion function in the neighborhood of its minimum helps to explain the behavior of the adjustment process at the approach to steady state. Figures 23 to 26 show profiles of $F(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ plotted versus one parameter at a time exhibiting the minimum, with the other parameters held fixed at the respective minimizing values. The curves pertain to the case of an analog model of known parameters, rather than to a human operator

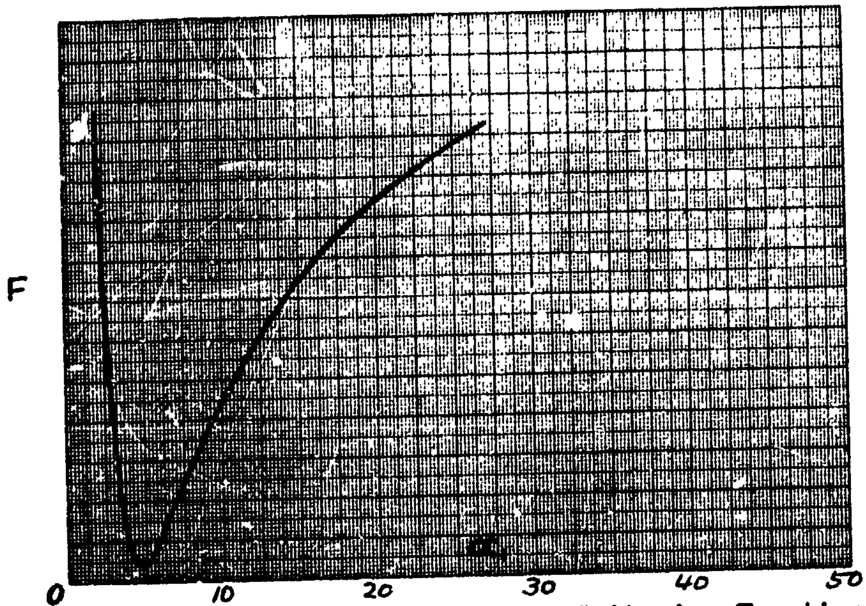


Figure No. 23 - Profile of the Criterion Function $F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$

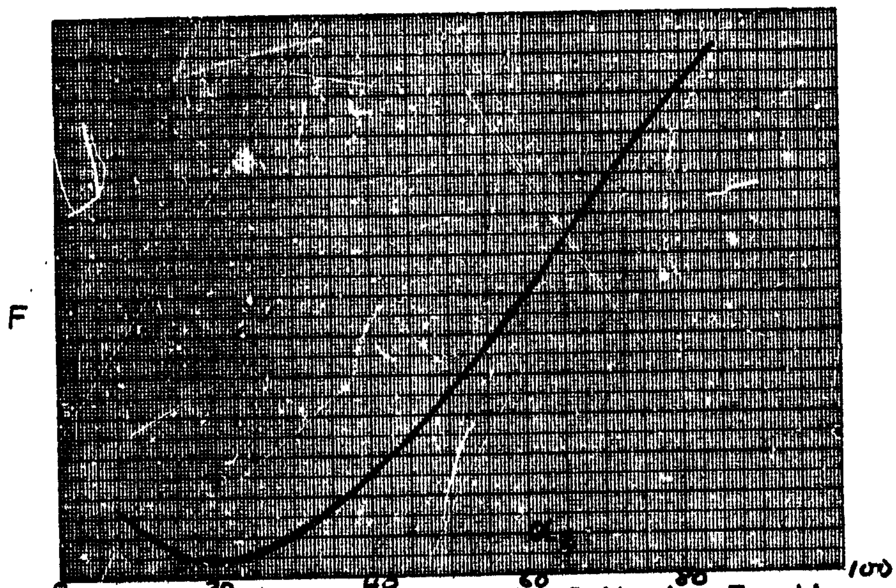


Figure No. 24 - Profile of the Criterion Function $F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$

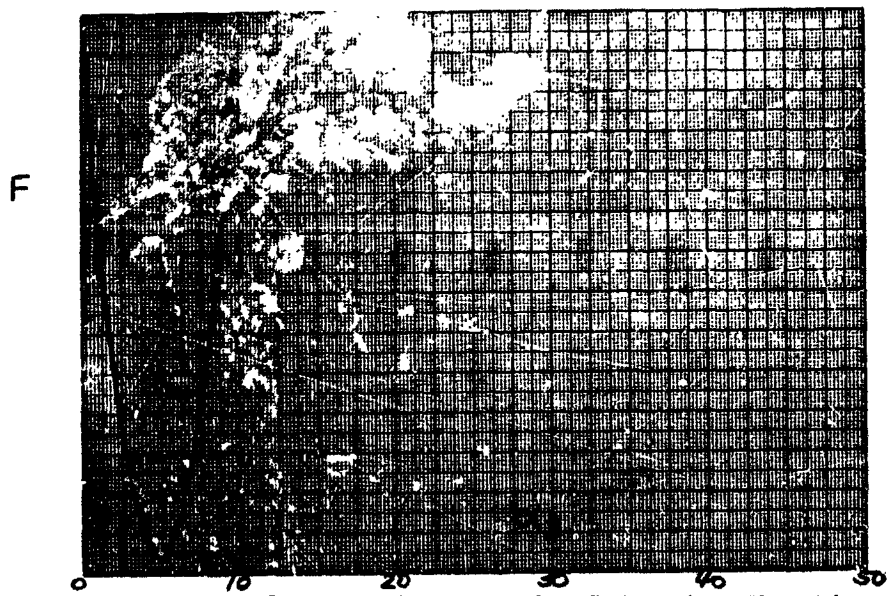


Figure No. 25 - Profile of the Criterion Function $F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$

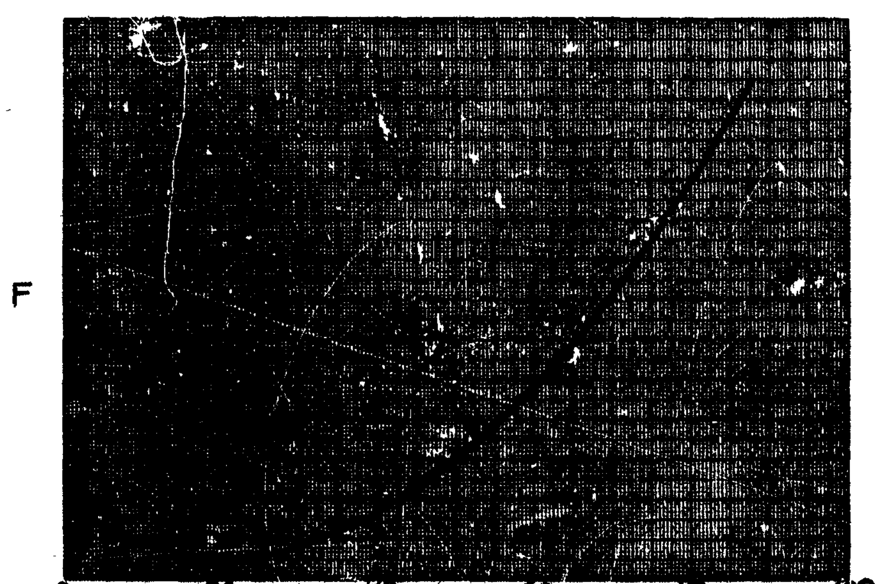


Figure No. 26 - Profile of the Criterion Function $F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$

and hence have a deterministic character. One observes that the minimum with respect to parameters α_2 and α_4 is quite flat, whereas the minimum is sharper in the case of parameters α_1 and α_3. This explains why the minimum-seeking process yields well-defined parameters α_1 and α_3 and poorly defined values α_2 and α_4, and hence explains the difference in the statistical variation of solutions exhibited in the scatter plots of experimental data (Figures 5 to 7). The continuous and step-by-step adjustment processes are affected by the nature of the minimum in an analogous manner, although the step-by-step adjustment is more sensitive to the shape of the minimum.

It should be noted that the profiles of Figures 23 to 26 are based on a known transfer function and not on actual human operators. Similar contours can be obtained for human tracking data and the above observations can be further verified.

5.5 Occurrence of Complex Roots

The results have shown (Table I) the presence of complex values for the parameters T_1 and T_2 occurring in the transfer function of equation (21). These complex roots did not occur in the work of Adams (Reference 4) since his model included a denominator term of the form $(1+\tau S)^2$ rather than a quadratic term. The occurrence of these complex roots is probably due to a lack of precision of the analog computation process in the vicinity of these roots. This low precision can be explained in two ways:

(1) Note that the complex roots T_1 and T_2 are obtained from factoring the quadratic

$$\frac{1}{\alpha_2} S^2 + \frac{\alpha_1}{\alpha_2} S + 1$$

and the profile of the criterion function F as a function of α_2 exhibits the flat minimum which makes high precision difficult to obtain.

(2) If one plotted F as a function of the parameter τ in Adams model, it would be seen that, at a double root the criterion function is flat to a higher order than at a single root. Hence the descent along the gradient will usually end at some threshold value of low slope which is located within an uncertainty region surrounding the exact root location. This property is familiar from numerical solutions of polynomials by gradient techniques (such as the Newton-Raphson Method) and from analog solution of polynomials as reported in the computer literature. The imaginary parts of the time constants T_2 and T_3 shown in Table 2 are small enough to be negligible for two of the three vehicle dynamics examined.

5.6 Mathematical Considerations.

Mathematical considerations favor the iterative and relaxation methods, since the adjustment process can be rigorously defined mathematically and convergence can be proved analytically. Specifically, the components of the gradient vector $\frac{\partial F}{\partial \alpha_1}$ can be computed exactly and requirements on step-size for obtaining a convergent sequence of values of the criterion function can be stated.

The continuous adjustment technique, on the other hand, suffers from a lack of mathematical rigor at the present time. The components of the gradient are not mathematically defined when the parameters are varying and the degree of approximation involved in the computation is unknown. Furthermore, the stability of the continuous adjustment process has not been proved in general, but only in certain specific cases. From this point of view, the step-by-step and iterative procedures can be considered more generally applicable since they require no a priori knowledge of system behavior, while the stability of the continuous method must be ascertained in each specific case by means of the analog computer.

As the steady-state is approached, the rate of change of the parameters become small and the gradient approximation becomes adequate for all practical purposes. This has been demonstrated in previous studies (Adams, Reference 4) and by the closely related work of Whittaker, et al, (Reference 6) and Margolis (Reference 7) where the same gradient approximation is used to implement model-adaptive control systems.

The stability of the continuous method has been studied analytically for specific cases by Margolis (Reference 7) and Donalson (Reference 8).

5.7 Suitability for On-Line Operation

The requirement for on-line parameter identification tends to favor the continuous method. If a sufficiently high rate of convergence can be established in the continuous adjustment process, the technique becomes suitable for on-line operation, yielding best-fit parameter values while the human operator continues to perform his tracking task.

The iterative method can be used for on-line operation only if several channels of recording and playback are available in conjunction with high-speed computers, capable of performing the parameter optimization faster than real time. Then the parameters corresponding to say a 15-second sample of data would be computed while the next 15-second sample was being recorded. The cost and complexity of this method may render it impractical for on-line use.

5.8 Questions Requiring Further Study

A number of questions arising during the program were left unanswered due to time limitations. The major problems are the following:

(a) Improvement of convergence time: The effect of such factors as the choice of criterion function on the speed of convergence needs further study. It may also be possible to increase the adjustment gain in the continuous method by use of stabilizing filters in the adjustment loop.

(b) Effect of finite difference approximation in computing the gradient. This problem has been mentioned under Section 5.3 above. The effect of using exact rather than approximate gradient components on the stability and convergence of the iterative method needs further study.

(c) Effect of additive noise: Each of the three methods was tested initially with a known transfer function. The effect of additive noise on the parameter adjustment process, however, was not studied. This is clearly an important consideration since some apparent "noise" or randomness exists in the human operator's output and the effect of this random component on the effectiveness of the model-matching techniques is not known.

(d) Improvement of continuous method: It has been mentioned above that the continuous method suffers in part from the fact that it cannot be defined mathematically due to the feedback problems involved. An alternate technique developed at STL is based on open-loop computation of increments to be added to the initial assumed parameter values. This method promises to avoid the mathematical difficulties of the continuous method while preserving its suitability to on-line operation. Furthermore, it results in a decrease in the number of multipliers required for implementation. This method has been studied to some extent analytically, but requires experimental verification.

An attempt will be made to include the above problems in the remaining portions of the research program.

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9. Adams, J. J., Unpublished Data on Human Transfer Function, NASA, Langley Research Center, March 1963. (Being prepared for publication).

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APPENDIX

COMPUTER IMPLEMENTATION

I - 1. Continuous Technique

This method which was described in Section 3.3 is closely related to the parameter adaptation methods used by Margolis (Reference 7), Whitaker et al. (Reference 6) and Adams (Reference 4). It uses the parameter influence programming technique developed by Meissinger (Reference 5).

The block diagram shown in Figure I-1 illustrates the computer implementation of the adjustment loop in principle. The differential equation of the human operator is assumed on the basis of prior knowledge of the operator's performance in the trading task. Continuous adjustment of the parameter values, starting from arbitrarily chosen initial settings, is effected by feedback signals generated in the parameter adjustment unit on the basis of a programmed model matching error criterion.

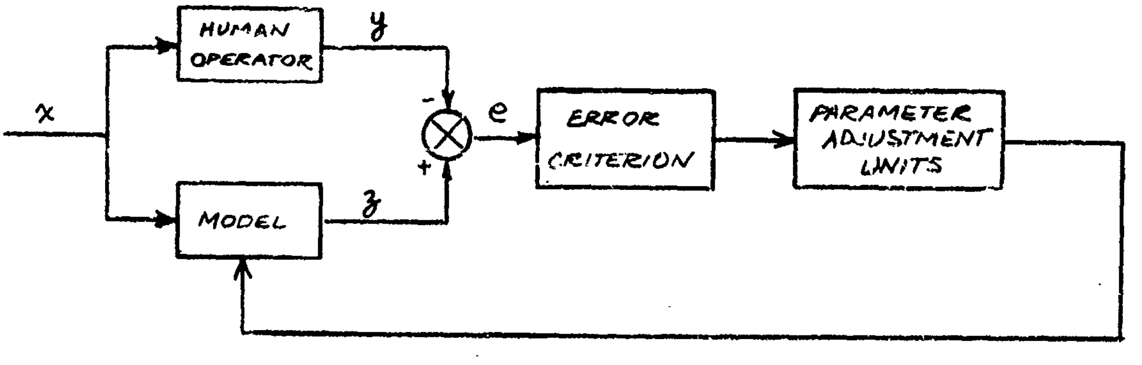


Figure I-1

The human operator is represented in this phase of the research study by a linear time-invariant differential equation with unknown fixed parameters $a_1, \ldots a_4$,

$$(I - 1) \qquad \ddot{y} + a_1 \dot{y} + a_2 y = a_3 \dot{x} + a_4 x$$

The initial values are $y(0) = c_1$, $\dot{y}(0) = c_2$

The model equation having adjustable parameters $\alpha_1, \ldots \alpha_4$ is defined by

$$(I - 2) \qquad \ddot{z} + \alpha_1 \dot{z} + \alpha_2 z = \alpha_3 \dot{x} + \alpha_4 x$$

The initial values are $z(0) = \gamma_1$, $\dot{z}(0) = \gamma_2$.

It is desired to match the parameters α_i of the model to those of the human operator equation by minimizing a specified error criterion F, which is a non-negative function of the model matching error

$$(I - 3) \qquad e = z - y$$

The criterion function is chosen to be

$$(I - 4) \qquad F = \frac{1}{2} e^2$$

The automatic process of minimizing $F = F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ approximates a steepest descent, or descent along the gradient $\left(\frac{\partial F}{\partial \alpha_1}, \frac{\partial F}{\partial \alpha_2}, \frac{\partial F}{\partial \alpha_3}, \frac{\partial F}{\partial \alpha_4}\right)$.

Steepest descent requires continuous or step-by-step parameter adjustment at a rate proportional to the local slope of the error criterion function.

$$(I - 5) \qquad \frac{d}{dt} \bar{\alpha} = - K \overline{\nabla F} \qquad K > 0$$

Gradient components $\frac{\partial F}{\partial \alpha_i}$ are expressed in terms of parameter influence coefficients $\frac{\partial z}{\partial \alpha_i}$ of the model output variable z. Using equations (I-4) and (I-5) one obtains

(I - 6) $$\frac{\partial F}{\partial \alpha_1} = e\frac{\partial e}{\partial \alpha_1} = e\,\frac{\partial z}{\partial \alpha_1}$$

Hence

(I - 7) $$\frac{d\alpha_i}{dt} = -Ke\frac{\partial z}{\partial \alpha_i}$$

To determine parameter influence coefficients on a continuous basis a set of additional differential equations must be programmed on the computer in the following manner. To simplify the notation let

(I - 8) $$u_i = \frac{\partial z}{\partial \alpha_i}$$

Equation (I-2) is formally differentiated with respect to α_1 whereby the order of differentiation with respect to t and α_1 can be interchanged. Thus, for example

(I - 9) $$\frac{\partial}{\partial \alpha_i}\left(\frac{\partial z}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial \alpha_i}\right) = \dot{u}_i$$

It is important to note that partial differentiation with respect to α_1 is a mathematically defined process only if α_1 is independent of time. Differentiation of equation (I-2) term by term with respect to α_1 using (I-9) results in

(I - 10) $$\ddot{u}_1 + \alpha_1 \dot{u}_1 + \alpha_2 u_1 = -\dot{z} \qquad \begin{array}{l} u_1(0) = 0 \\ \dot{u}_1(0) = 0 \end{array}$$

The forcing term x and $\dot{x}$ being independent of α_1 do not appear in (I-10). This equation, known as sensitivity equation must be solved simultaneously with equation (I-2) to yield the parameter influence coefficient $\frac{\partial z}{\partial \alpha_1}$ and hence the gradient component $\frac{\partial F}{\partial \alpha_1}$. The other components are obtained similarly by programming and solving additional sensitivity equations with respect to α_2, α_3, α_4.

The circuit diagrams (2), (3), (4), (5) in Figure 1-A show the computer program for obtaining the partial derivatives. The notation used in this diagram has been changed as follows, to convert the equations into first order form:

$$(\text{I - 11}) \qquad \begin{aligned} z &\triangleq z_1 \\ \dot{z} &\triangleq z_2 \\ x &\triangleq x_1 \\ \dot{x} &\triangleq x_2 \end{aligned} \qquad\qquad \begin{aligned} \frac{\partial z}{\partial \alpha_1} &\triangleq \frac{\partial z_1}{\partial \alpha_1} = u_{11} \\ \frac{\partial \dot{z}}{\partial \alpha_1} &\triangleq \frac{\partial z_2}{\partial \alpha_2} = u_{21} \\ &\vdots \end{aligned}$$

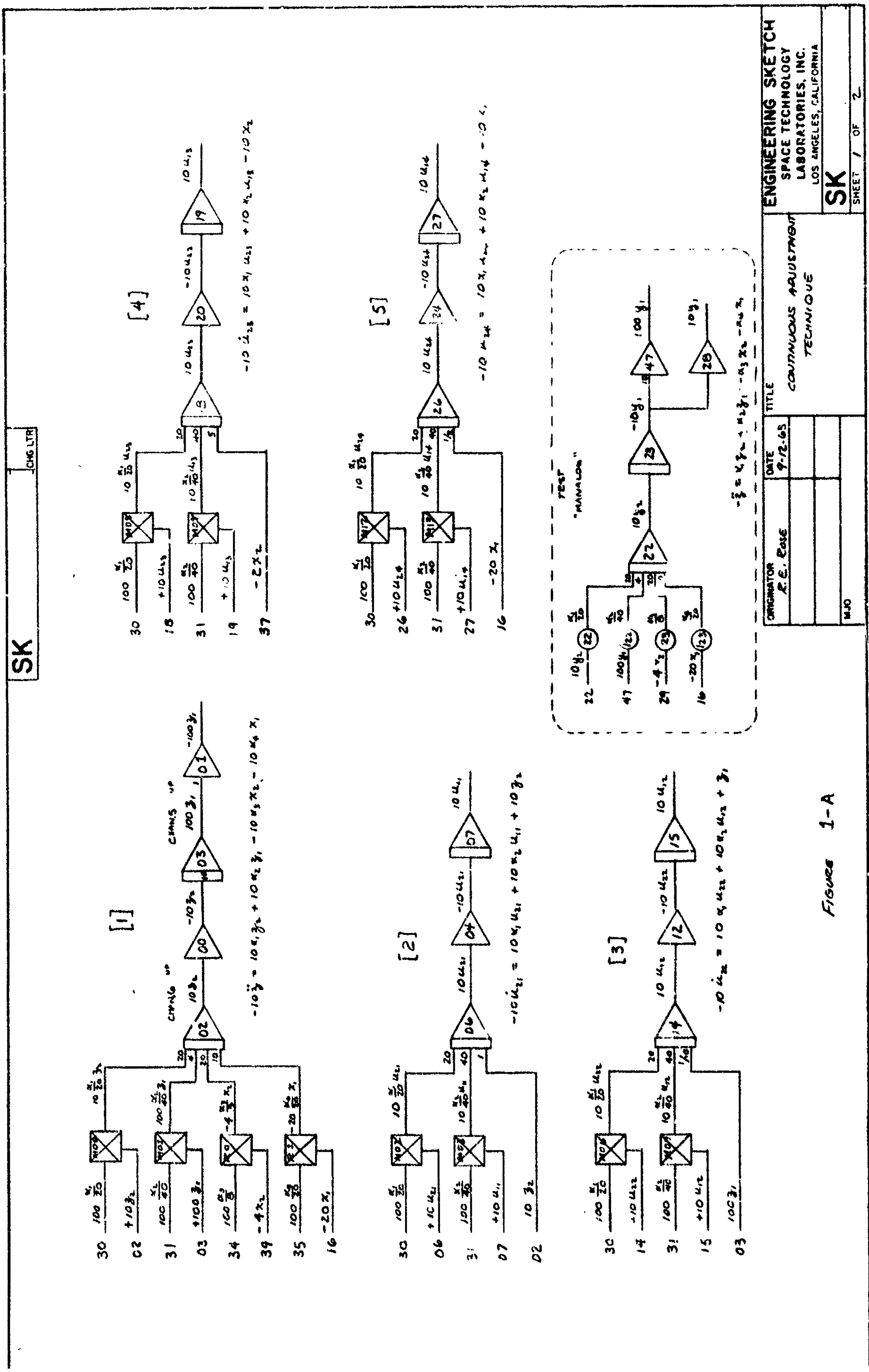
The sensitivity equations can now be expressed in the form

$$(\text{I - 12}) \qquad \begin{aligned} \dot{u}_{21} + \alpha_1 u_{21} + \alpha_2 u_{11} &= -z_2 \\ \dot{u}_{22} + \alpha_1 u_{22} + \alpha_2 u_{12} &= -z_1 \\ \dot{u}_{23} + \alpha_1 u_{23} + \alpha_2 u_{13} &= x_2 \\ \dot{u}_{24} + \alpha_1 u_{24} + \alpha_2 u_{14} &= x \end{aligned}$$

Circuit (11) in Figure 2-A is used to obtain the desired rate of change of parameters α_1 according to equation (I-7). These terms are then integrated and fed back to adjust the parameter settings in the original model equation (I-2) and the sensitivity equations (I-12).

The disturbance signals and the human operator output signals are obtained from previously recorded tapes and fed to the computer circuits (7), (9), and (12). Circuit (8) is used to compute the derivative $\dot{x} = x_2$ which is required to implement the forcing function in the third equation of (I-12)

Each of the partial derivatives are found using circuits (2), (3),


ENGINEERING SKETCH
SPACE TECHNOLOGY LABORATORIES, INC.
LOS ANGELES, CALIFORNIA
SK
SHEET 1 OF 2
TITLE
CONTINUOUS ADJUSTMENT TECHNIQUE
ORIGINATOR R.E. ROSE
DATE 9-12-65
TEST "ANALOG"
FIGURE 1-A

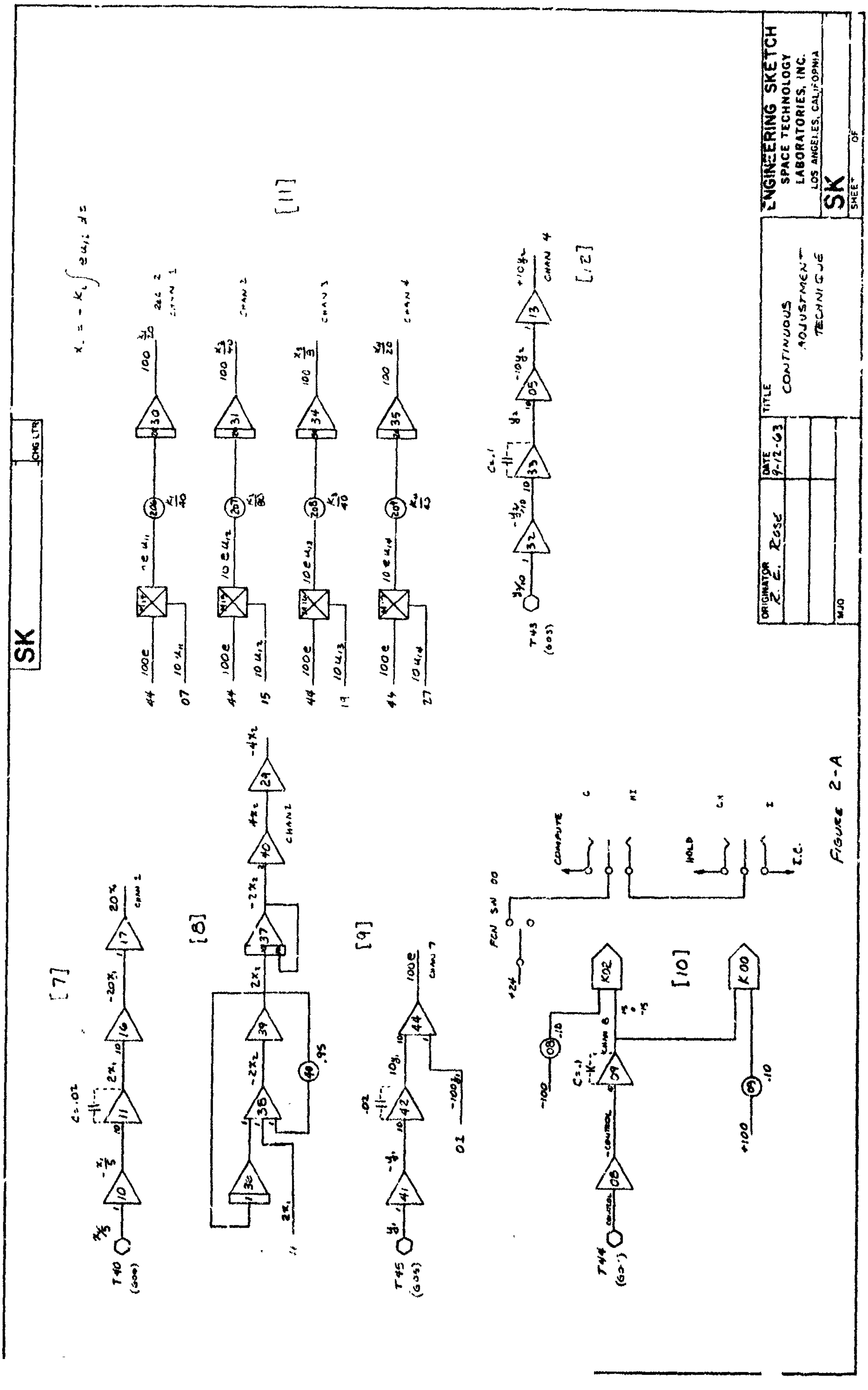

ENGINEERING SKETCH
SPACE TECHNOLOGY LABORATORIES, INC.
LOS ANGELES, CALIFORNIA
SK
TITLE
CONTINUOUS ADJUSTMENT TECHNIQUE
ORIGINATOR
R. E. ROSE
DATE
9-12-63
COMPUTE
HOLD
I.C.
FCN SW 00

FIGURE 2-A

(4) and (5). These are used in equations (I-12) and circuit (11) to obtain the parameters. Circuit (10) employs a signal which was taped on one channel of the tape recorder to control the computer operating mode.

The mathematical problem inherent in this approach to gradient computation is that the α_i terms were assumed to be time-invariant in the derivation, as noted above. Actually, the continuous adjustment process violates this condition. Thus the gradient components are determined only approximately. The approximation error is not known, but depends strongly on the rate of adjustment.

It is also interesting to note that linearity and time-invariance of the model differential equation is not a prerequisite for implementation of the (approximate) gradient computation. In the case of time-variant or non-linear differential equations the structure of the sensitivity equations no longer resembles that of the original differential equation and the computer programming becomes somewhat more complex.

I - 2. Iterative Technique

The iterative model matching technique was concerned with the objective of circumventing the mathematical difficulty associated with gradient computation in the continuous adjustment technique. All parameters are held constant during the time interval of gradient computation. This eliminates the (unknown) approximation error of the gradient due to a mathematically inexact process of determining sensitivity functions.

A further departure in computer programming from the previous technique is the use of finite difference approximation for finding the desired partial derivatives $\frac{\partial z}{\partial \alpha_i}$. The block diagram (Figure I-2) shows two models operating at slightly different settings of the parameter α_i. The first model yields the output signal z. The perturbed model solving equation (I-2) for the parameter value $\alpha_i + \Delta\alpha_i$ yields an output signal ζ. The partial derivative is thus approximated by

(I - 13) $$\frac{\partial z}{\partial \alpha_i} \cong \frac{\Delta z}{\Delta \alpha_i} = \frac{z(\alpha_i) - z(\alpha_i + \Delta\alpha_i)}{\Delta \alpha_i} = \frac{z - \zeta}{\Delta \alpha_i}$$

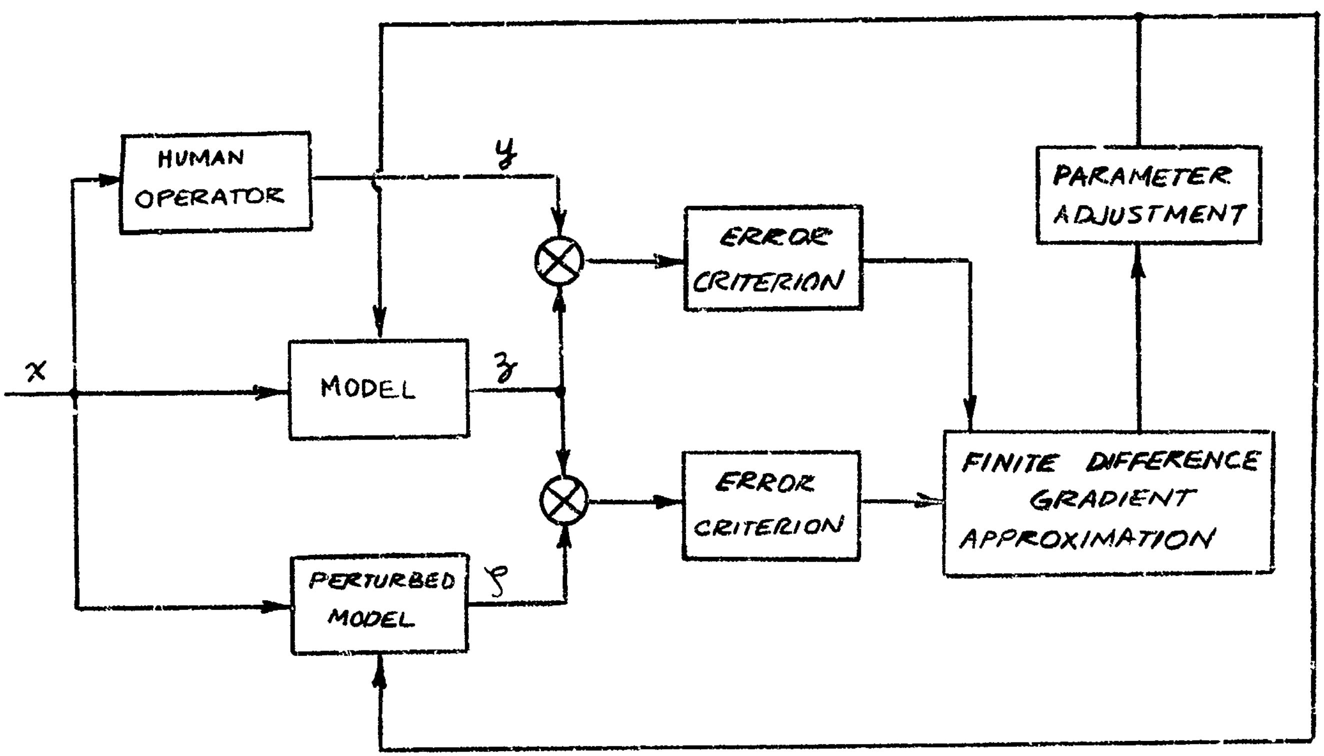


Figure I-2

It is seen that one perturbed model is necessary for each parameter being adjusted, each perturbed model having a different α_i increased by $\Delta\alpha_i$. Considerable savings in computer equipment can be achieved by employing only one perturbed model for all $\Delta\alpha_i$ plus switching circuits which apply $\Delta\alpha_i$ to each parameter of interest in turn, one at a time.

EQUATIONS FOR COMPUTER SOLUTION

The following equations are those implemented on the computer. The differential equation of the model is given by

(I - 14) $$\ddot{z} + \alpha_1^{(K)} \dot{z} + \alpha_2^{(K)} z = \alpha_3^{(K)} \dot{x} + \alpha_4^{(K)} x$$

where the superscript (K) refers to the value of a paramter after the Kth iteration. The perturbed model equation is

$$\ddot{\xi} + \left\{\alpha_1^{(K)} + \Delta\alpha_1 \; \delta\left[\sin\left(\tfrac{K-1}{4}\right)\pi\right]\right\}\dot{\xi} + \left\{\alpha_2^{(K)} + \Delta\alpha_2 \; \delta\left[\sin\left(\tfrac{K-2}{4}\right)\pi\right]\right\}\xi =$$

(I - 15) $$\left\{\alpha_3^{(K)} + \Delta\alpha_3 \; \delta\left[\sin\left(\tfrac{K-3}{4}\right)\pi\right]\right\}\dot{x} + \left\{\alpha_4^{(K)} + \Delta\alpha_4 \; \delta\left[\sin\left(\tfrac{K-4}{4}\right)\pi\right]\right\}x$$

The symbol

$$\delta\left[\sin\left(\frac{K-n}{4}\right)\pi\right]$$

is the Kronecker delta which is employed here as notation to indicate which parameter is being selected for perturbation at any given time.

The computer circuit diagrams are numbered for reference. Circuits (1) and (2) are the input quantities from the tape recorder, the disturbance function and the human operator output, and their derivatives respectively. Circuits (3) and (4) are the model and the perturbed model respectively (see Figure 1-B)

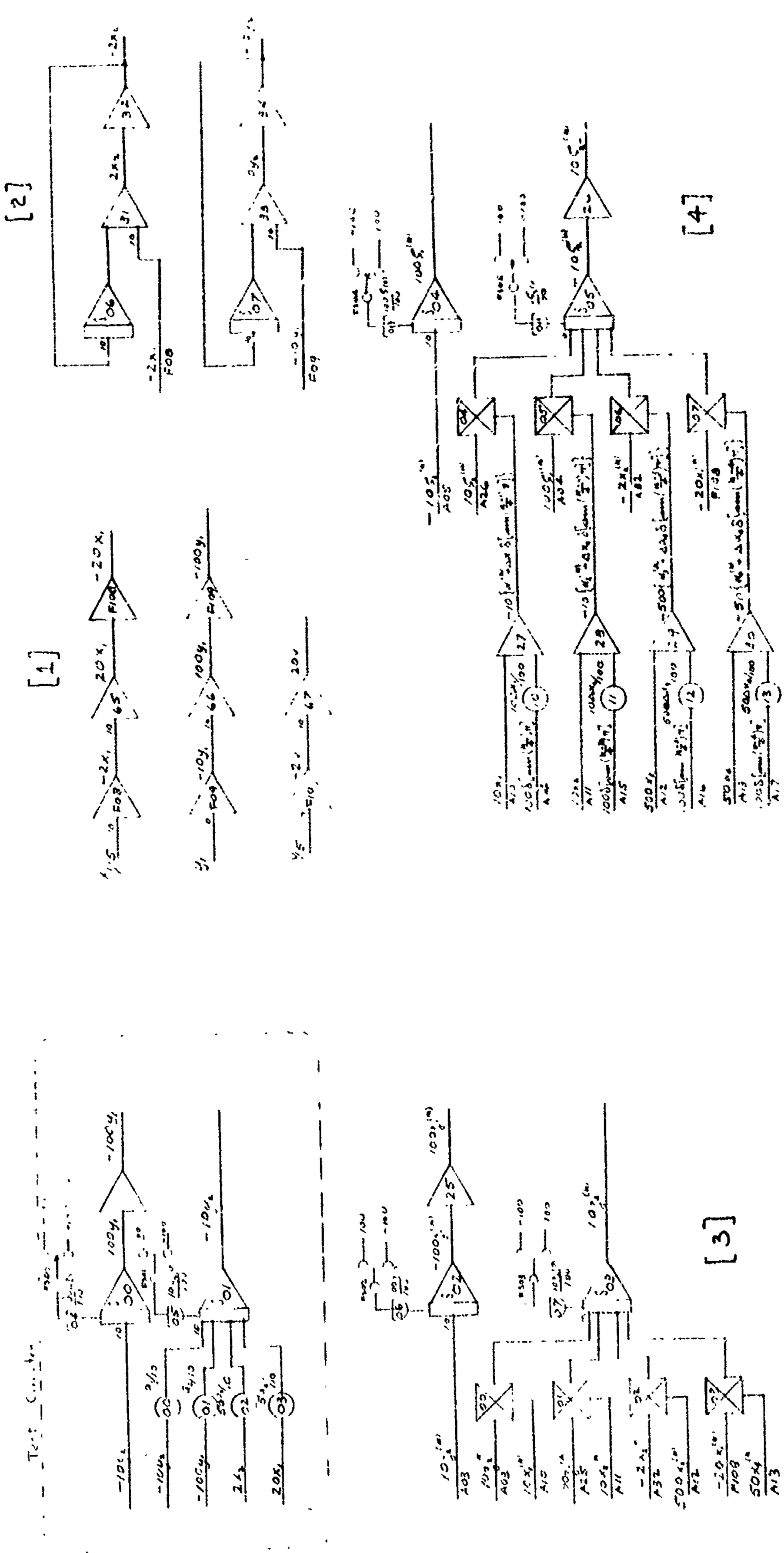


FIGURE 1-B

Iterative Method 7/1/63

[5]

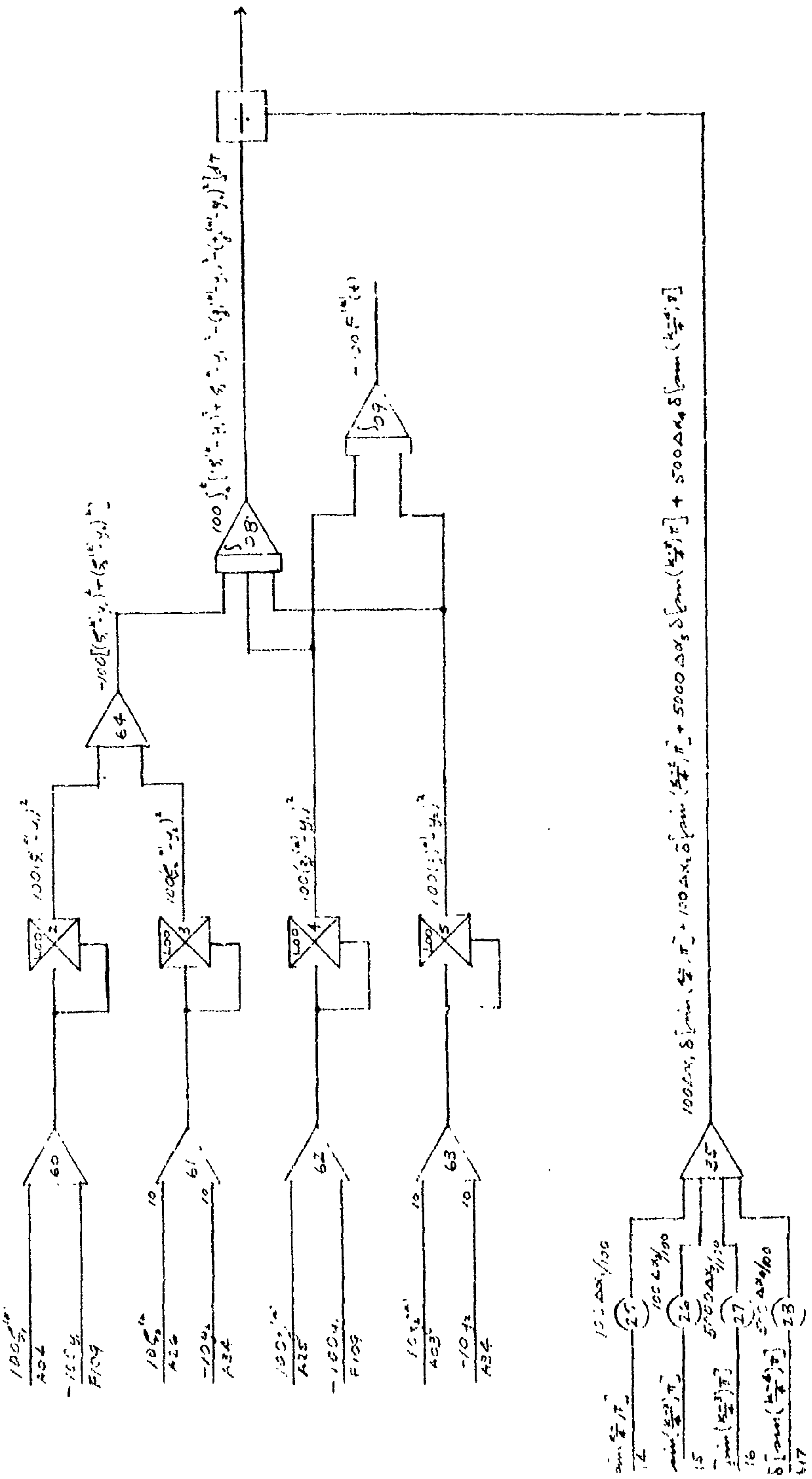


FIGURE 2-B

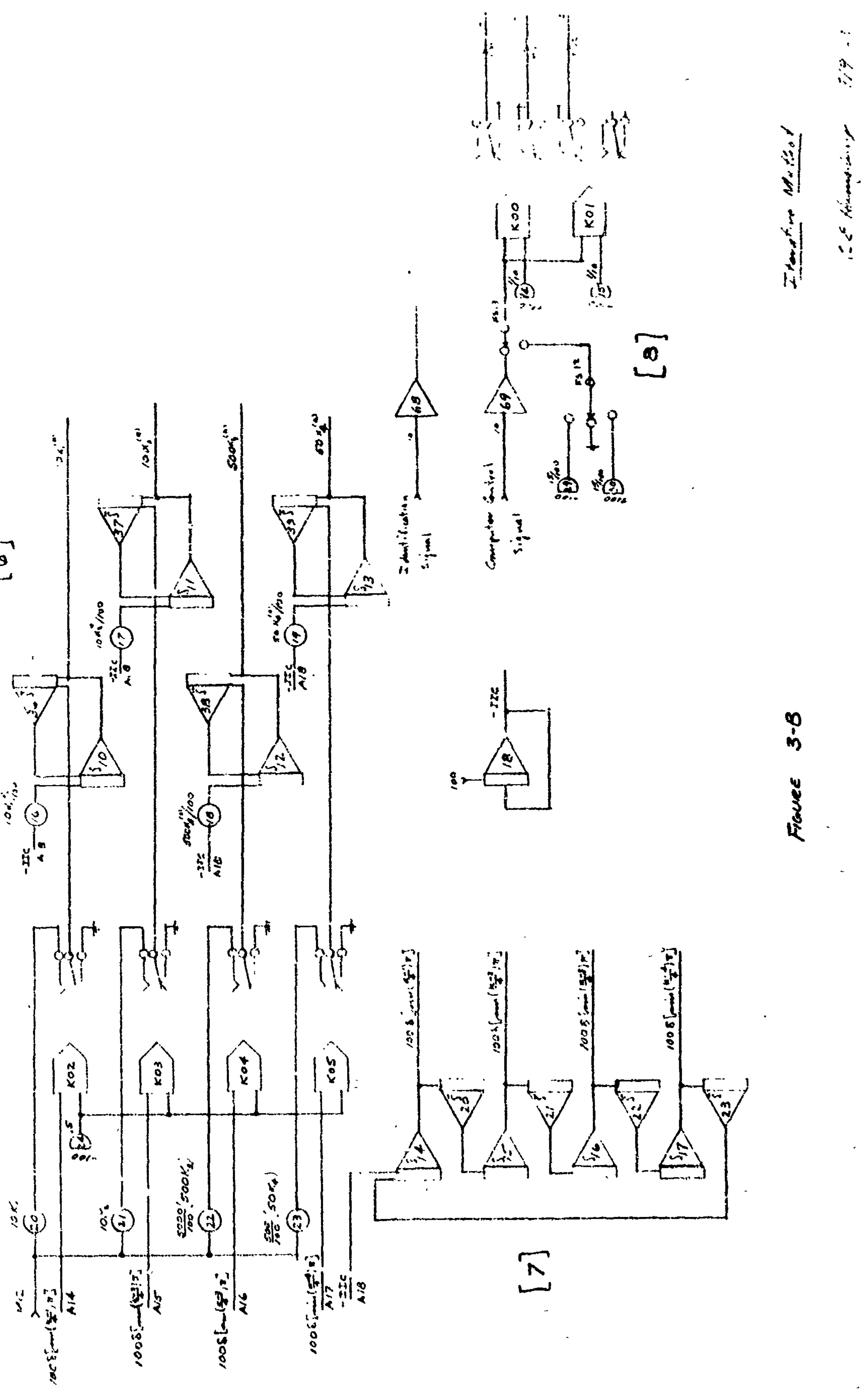


FIGURE 3-B

The error criterion selected is given by

(I - 16) $$F = \int_0^T \left(e^2 + \dot{e}^2 \right) dt$$

The error criterion involving the human operator output versus model output may be expressed by

(I - 17) $$F(\alpha_i) = \int_0^T \left[\left(z - y \right)^2 + \left(\dot{z} - \dot{y} \right)^2 \right] dt$$

on the other hand, the error criterion involving operator output versus perturbed model output is

(I - 18) $$F\left(\alpha_i + \Delta \alpha_i\right) = \int_0^T \left[\left(\zeta - y \right)^2 + \left(\dot{\zeta} - \dot{y} \right)^2 \right] dt$$

Thus the finite difference quotient approximating $\frac{\partial F}{\partial \alpha_i}$ is

(I - 19) $$Q^{(K)}(t) = \frac{\int_0^T \left[(\zeta - y)^2 + (\dot{\zeta} - \dot{y})^2 \right] dt - \int_0^T \left[(z - y)^2 + (\dot{z} - \dot{y})^2 \right] dt}{\Delta \alpha_i}$$

This quantity is computed by circuit (5). (See Figure 2-B).

Circuit (6) is a memory circuit which stores the values of the α's and, after any new α_i has been calculated, stores this new α_i. The equation describing this process is

(I - 20) $$\alpha_i^{(K+1)} = \alpha_i^{(K)} + \Delta\alpha_i^{(K)}$$

where

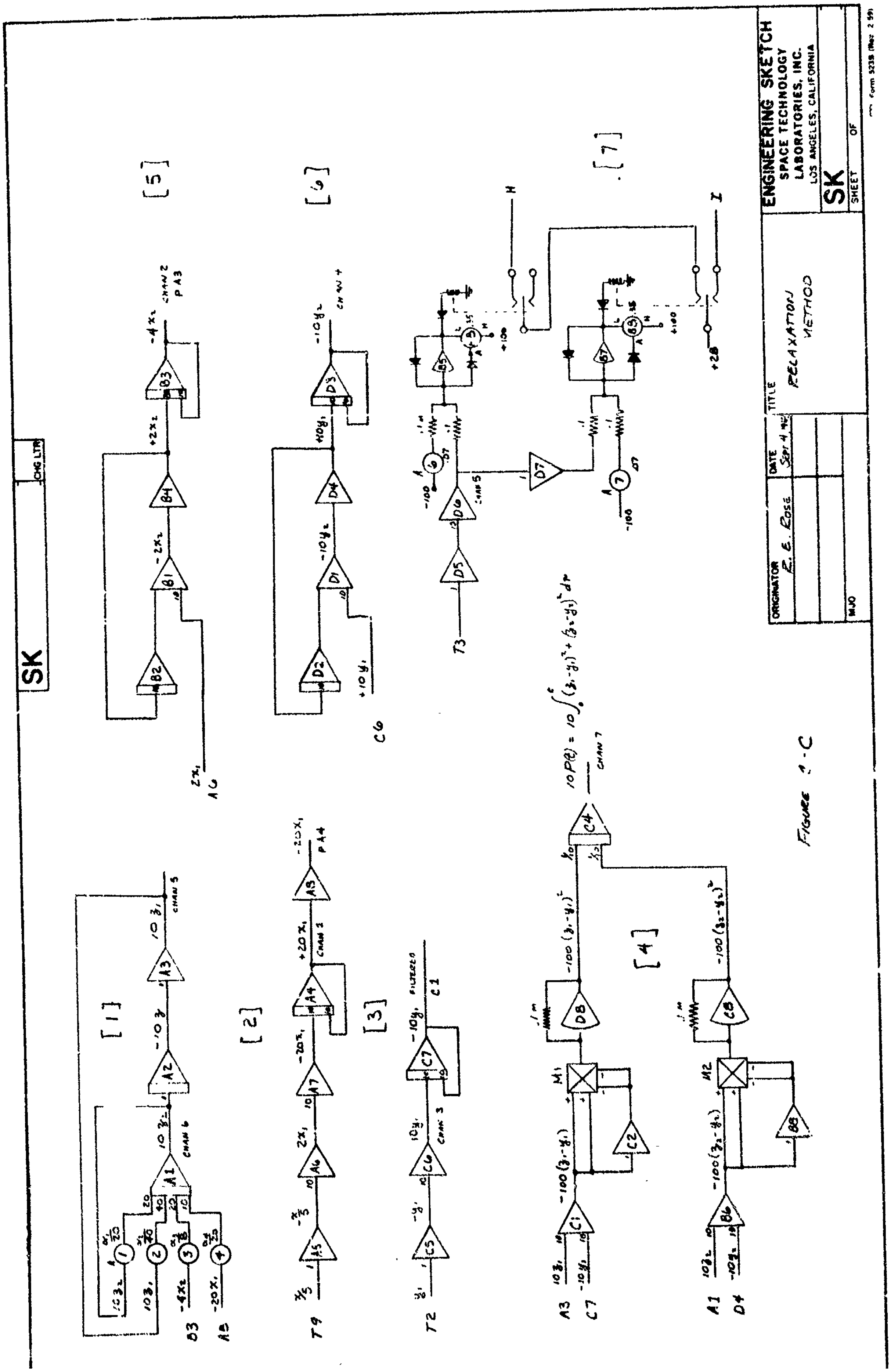
(I - 21) $$\Delta \alpha_i^{(K)} = - K \, \delta \left[\sin \left(\frac{K - i}{4} \right) \pi \right] Q^{(K)}(T)$$

Circuit (7) generates the Kronecker delta employed in equations (2) and (9). The four outputs of the circuit energize cyclically as the parameters are adjusted. Circuit (8) controls the computer mode as prescribed by a previously recorded signal (see Figure 3-B).

I - 3. Relaxation Technique

The relaxation technique employed was simplified greatly by eliminating the automatic parameter adjustment loop. It requires only the formulation of a model and an error criterion function which is minimized by manually adjusting potentiometers. The method employed is that described in Section 3.3

The computer diagram has been numbered for reference. Circuits (2), (3), (5) and (6) are input gain circuits and derivatives, respectively. Circuit (7) controls the modes of the computer. (See Figure 1-C).


ENGINEERING SKETCH
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SHEET
OF
TITLE
RELAXATION METHOD
ORIGINATOR
R. E. Rose
DATE
Sept 4 '62
CHG LTR
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CHAN 5
CHAN 6
CHAN 2
CHAN 3
CHAN 7
PA 3
PA 4
FILTERED
$10P(t) = 10\int_0^t (z_1-y_1)^2 + (z_2-y_2)^2 \, dt$
$-100(z_1-y_1)^2$
$-100(z_2-y_2)^2$
$-100(z_1-y_1)$
$-100(z_2-y_2)$
FIGURE 1-C